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Remarks on the Validity of the Cottingham Formula for Electromagnetic Mass Shifts*

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We discuss the effect of Schwinger terms on the Wick rotation necessary to derive the Cottingham formula for electromagnetic mass shifts. We give a simple condition which allows us to construct a covariant and gauge-invariant virtual Compton scattering amplitude on which the Wick rotation can be performed.

The Cottingham formula[1] relating the electromagnetic self-mass δM of hadrons to integrals over the forward virtual spin-averaged Compton scattering amplitudes[2] $T^*_{\mu\nu}(\nu, q^2)$ for spacelike values of q^2 only has been widely used to calculate electromagnetic mass shifts of hadrons.[3] It has also been the starting point of discussions relating to the divergences which appear in electromagnetic self-mass calculations.[4] In view of this interest we would like to comment in this paper on the validity of the Cottingham formula.[5] We will prove the following theorem.

Theorem. Define

$$h_{\mu\nu}(x,p)=\langle p|[j_\mu(x),j_\nu(0)]|p\rangle,$$

where $j_\mu(x)$ is the Heisenberg electromagnetic current operator and $|p\rangle$ is a single-hadron state of momentum p. If $h_{\mu\nu}(x,p)\delta(x_0)$ is well defined[6] and $h_{00}(x,p)\delta(x_0)=0$ in all Lorentz frames, then it is possible to find a covariant and gauge-invariant $T^*_{\mu\nu}(\nu,p)$ such that the contour rotation necessary to get the Cottingham formula is valid and the energy integral is finite if the rotation is performed before doing the spatial-momentum integrals.

We will critically examine our assumption $h_{00}(x,p)\delta(x_0)=0$ and show that, although the assumption is plausible, it is possible to construct simple forms for $h_{\mu\nu}(x,p)\delta(x_0)$ which are consistent with general symmetry requirements and which have the property that $h_{00}(x,p)\delta(x_0)\neq 0$. We will also discuss briefly the order of integrations for the spatial and time components and indicate that the conditions of the theorem do not imply that the mass shift is finite.

Lemma 1. If $h_{00}(x,p)\delta(x_0)=0$ in all Lorentz frames, then $h_{0i}(x,p)\delta(x_0)$ and $h_{ij}(x,p)\delta(x_0)$ contain at most one derivative of a δ function. A slightly less general version of this lemma has been proved by Gupta and Rajasekaran.[7]

Proof. The statement that $h_{00}(x,p)\delta(x_0)=0$ is true in all Lorentz frames implies that $n_\mu n_\nu h_{\mu\nu}(x,p)\times\delta(n\cdot x)=0$, for all n_λ timelike. Differentiating this with respect to n_λ and using the constraint imposed by current conservation on $h_{\mu\nu}(x,p)$, we get

$$(g_{ij}+\partial_j x_i)\delta(x_0)h_{0j}(x,p)=0\,. \tag{1}$$

Assuming that $h_{\mu\nu}(x,p)\delta(x_0)$ is well defined, the most general form for $\delta(x_0)h_{0j}(x,p)$ is

$$\delta(x_0)h_{0j}(x,p)=\sum_{m,n}C_{mn}\,\partial_j(\partial\cdot p-\partial_0 p_0)^m(\Box-\partial_0{}^2)^n\delta^4(x) + \sum_{m,n}D_{mn}\,p_j(\partial\cdot p-\partial_0 p_0)^{m+1}(\Box-\partial_0{}^2)^n\delta^4(x), \tag{2}$$

where we assume a finite number of terms. Substituting (2) into (1) gives

$$(m+2n)C_{mn}=0,\qquad (m+2n)D_{mn}=0,\quad \text{with } m,n\geq 0 \tag{3}$$

which gives $C_{mn}=D_{mn}=0$, unless $m=0$, $n=0$ and thus proves the lemma for $\delta(x_0)h_{0i}(x,p)$. To complete the proof of the lemma we must show that $\delta(x_0)h_{ij}(x,p)$ has no derivatives of δ functions higher than one. We have just shown that

$$x_\alpha x_\beta\delta(n\cdot x)n_\lambda h_{\lambda\nu}(x,p)=0\,. \tag{4}$$

Differentiating with respect to n_μ, using the constraint of current conservation on $h_{\mu\nu}(x,p)$, and then setting $n_\lambda=g_{\lambda 0}$ gives

$$x_\alpha x_\beta\delta(x_0)h_{\mu\nu}(x,p)+x_\alpha x_\beta x_\mu\partial_0 h_{0\nu}(x,p)\delta(x_0) - x_\alpha x_\beta x_\mu\delta(x_0)\partial_k h_{k\nu}(x,p). \tag{5}$$

Setting $\alpha=l$, $\beta=m$, $\mu=i$, and $\nu=j$, we get

$$x_l x_m\partial_k[x_i\delta(x_0)h_{kj}]=0. \tag{6}$$

Substituting the general form

$$\delta(x_0)h_{kj}(x,p)=\sum_{m,n}[C_1^{mn}p_k p_j(\partial\cdot p-\partial_0 p_0)^m+C_2^{mn}(p_k\partial_j+p_j\partial_k)(\partial\cdot p-\partial_0 p_0)^{m-1}+C_3^{mn}\partial_k\partial_j(\partial\cdot p-\partial_0 p_0)^{m-2}](\Box-\partial_0{}^2)^n\delta^4(x) \tag{7}$$

in this equation it is easy to establish the lemma.

Lemma 2. Under the conditions of Lemma 1 the most general form for $h_{\mu\nu}(x,p)\delta(x_0)$ is

$$\begin{aligned}&\delta(x_0)h_{00}(x,p)=0,\\ &\delta(x_0)h_{0i}(x,p)=i[C(p_0)\partial_i-D(p_0)p_i(\partial\cdot p-\partial_0p_0)]\delta^4(x),\\ &\delta(x_0)h_{ij}(x,p)=i[-D(p_0)(\partial_jp_i+\partial_ip_j)-C'(p_0)g_{ij}(\partial\cdot p-\partial_0p_0)+D'(p_0)p_ip_j(\partial\cdot p-\partial_0p_0)]\delta^4(x),\end{aligned}\tag{8}$$

where $C(p_0)$ and $D(p_0)$ are arbitrary real functions of p_0 and the primes denote differentiation with respect to p_0.

Proof. This result has been established by Creutz and Sen in another discussion.[8] However, for the sake of completeness we give a sketch of the proof.

General symmetry principles [e.g., translation invariance, Hermiticity of $j_\mu(x)$, CTP] imply $h_{\mu\nu}(x,p)=-h_{\mu\nu}(-x,p)$, i.e., the equal-time commutator can contain only terms with an odd number of derivatives on a spatial δ function. Consistent with Lemma 1 the most general form for $h_{\mu\nu}(x,p)$ would be

$$\sqrt{n^2}\,\delta(n\cdot x)h_{\mu\nu}(x,p)=iC_{\mu\nu\alpha}(n/\sqrt{n^2},p)\partial_\alpha\delta^4(x),\tag{9}$$

where n_μ is a timelike vector with $n_0>0$. If $h_{\mu\nu}(x,p)$ is to have a well-defined equal-time limit the combinations of derivatives in (9) must involve no time derivatives in a frame where n has only a time derivative. This means we must require

$$n_\alpha C_{\mu\nu\alpha}(n/\sqrt{n^2},p)=0.\tag{10}$$

The symmetry conditions require

$$C_{\mu\nu\alpha}=C_{\nu\mu\alpha}\,.\tag{11}$$

Current conservation, $\partial_\mu h_{\mu\nu}(x,p)=\partial_\nu h_{\mu\nu}(x,p)=0$, gives

$$\begin{aligned}C_{\mu\nu\alpha}&=-\sqrt{n^2}\,\frac{d}{dn_\alpha}\left(\frac{n_\lambda}{\sqrt{n^2}}C_{\lambda\nu\mu}\right),\\ C_{\mu\nu\alpha}&=-\sqrt{n^2}\,\frac{d}{dn_\alpha}\left(\frac{n_\lambda}{\sqrt{n^2}}C_{\mu\lambda\nu}\right).\end{aligned}\tag{12}$$

Taking the difference of these equations we find that the quantity

$$\frac{n_\lambda}{\sqrt{n^2}}C_{\lambda\nu\mu}-\frac{n_\lambda}{\sqrt{n^2}}C_{\mu\lambda\nu}\tag{13}$$

cannot depend on n. Because of (11) this quantity is antisymmetric under interchange of μ and ν. Thus,

$$C_{\mu\nu}=\frac{n_\lambda}{\sqrt{n^2}}C_{\mu\lambda\nu}=\frac{n_\lambda}{\sqrt{n^2}}C_{\lambda\nu\mu}\,,$$

with

$$n_\mu C_{\mu\nu}=n_\nu C_{\mu\nu}=0.\tag{14}$$

Hence,

$$C_{\mu\nu\alpha}=-\sqrt{n^2}\,\frac{d}{dn_\alpha}C_{\mu\nu}\,.\tag{15}$$

Finally, note that any $C_{\mu\nu}(n/\sqrt{n^2},p)$ satisfying (14) will define through (15) a $C_{\mu\nu\alpha}$ which will satisfy the constraints of current conservation. This means the most general form for $h_{\mu\nu}(x,p)$ at equal times is given by

$$\sqrt{n^2}\,\delta(n\cdot x)h_{\mu\nu}(x,p)=i\sqrt{n^2}\left(\frac{d}{dn}\cdot\frac{d}{dx}\right)C_{\mu\nu}\left(\frac{n}{\sqrt{n^2}},p\right)\delta^4(x),\tag{16}$$

with

$$\begin{aligned}C_{\mu\nu}\left(\frac{n}{\sqrt{n^2}},p\right)=&-C\left(\frac{p\cdot n}{\sqrt{n^2}}\right)\left(g_{\mu\nu}-\frac{n_\mu n_\nu}{n^2}\right)\\ &+D\left(\frac{p\cdot n}{\sqrt{n^2}}\right)\left(p_\mu-\frac{n_\mu p\cdot n}{n^2}\right)\left(p_\nu-\frac{n_\nu p\cdot n}{n^2}\right).\end{aligned}\tag{17}$$

Carrying out the differentiation in (16), we can obtain the explicit form for the equal-time commutator in terms of C and D given in the lemma.

Lemma 3. If $T_{\mu\nu}(\nu,q^2)$ is defined by

$$T_{\mu\nu}(\nu,q^2)=2i\int d^4x\,e^{iq\cdot x}\theta(x_0)\langle p|[j_\mu(x),j_\nu(0)]|p\rangle,\tag{18}$$

then under the conditions of Lemmas 1 and 2,[9] $T^*_{\mu\nu}(\nu,q^2)$ defined by

$$\begin{aligned}T^*_{\mu\nu}(\nu,q^2)=T_{\mu\nu}(\nu,q^2)&+2C(p_0)(g_{\mu\nu}-g_{\mu0}g_{\nu0})\\ &-2D(p_0)(p_\mu-p_0g_{\mu0})(p_\nu-p_0g_{\nu0})\end{aligned}\tag{19}$$

is covariant and gauge-invariant.

Proof. Following Bjorken,[10] we assume that $T^*_{\mu\nu}$ and $T_{\mu\nu}$ considered as analytic functions of q_0 have the same absorptive parts so that they differ at most by polynomials in q_0.

Since we wish to construct a covariant and gauge-invariant $T^*_{\mu\nu}(\nu,q^2)$, we write

$$\begin{aligned}T^*_{\mu\nu}(\nu,q^2)=&T^*_1(\nu,q^2)\left(g_{\mu\nu}-\frac{q_\mu q_\nu}{q^2}\right)\\ &+T^*_2(\nu,q^2)\left(p_\mu-q_\mu\frac{p\cdot q}{q^2}\right)\left(p_\nu-q_\nu\frac{p\cdot q}{q^2}\right),\end{aligned}\tag{20}$$

while for $T_{\mu\nu}(\nu,q^2)$ we have to write

$$\begin{aligned}T_{\mu\nu}=&T_1g_{\mu\nu}+T_2p_\mu p_\nu+T_3q_\mu q_\nu+T_4n_\mu n_\nu+T_5(p_\mu q_\nu+p_\nu q_\mu)\\ &+T_6(p_\mu n_\nu+p_\nu n_\mu)+T_7(q_\mu n_\nu+q_\nu n_\mu),\end{aligned}\tag{21}$$

when $n_\mu = g_{\mu 0}$. Here the T_i are functions of q^2, ν, $q\cdot n$, $p\cdot n$. Because T_1, T_2 differ from T_1^*, T_2^* by polynomials in $q_0 = q\cdot n$, it follows that T_4, T_6, and T_7 are polynomials in q_0.

From the definition of $T_{\mu\nu}$ using current conservation it follows that

$$q_\mu T_{\mu\nu} = -2\int d^4x\, e^{iq\cdot x} n_\mu \delta(n\cdot x) h_{\mu\nu}(x,p). \tag{22}$$

Using the form for $h_{\mu\nu}(x,p)\delta(n\cdot x)$ given in Lemma 2 it is easy to show that

$$\begin{aligned} T_3 &= -\frac{1}{q^2}T_1 + T_2\frac{(p\cdot q)^2}{q^4} - \frac{2}{q^2}C(p_0) - 2\frac{(p\cdot q)^2}{q^4}D(p_0),\\ T_4 &= 2C(p_0) + 2p_0^2 D(p_0),\\ T_5 &= -T_2\frac{p\cdot q}{q^2} + 2\frac{p\cdot q}{q^2}D(p_0),\\ T_6 &= -2D(p_0)p_0,\\ T_7 &= 0. \end{aligned} \tag{23}$$

Therefore, we have

$$\begin{aligned} S_{\mu\nu} = T_{\mu\nu} - T^*_{\mu\nu} = (T_1 - T_1^*)\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) + (T_2 - T_2^*)\left(p_\mu - q_\mu\frac{p\cdot q}{q^2}\right)\left(p_\nu - q_\nu\frac{p\cdot q}{q^2}\right) + 2C(p_0)\left(-\frac{1}{q^2}q_\mu q_\nu + g_{\mu 0}g_{\nu 0}\right)\\ + 2D(p_0)\left[-\frac{(p\cdot q)^2}{q^4}q_\mu q_\nu + p_0^2 g_{\mu 0}g_{\nu 0} - p_0(p_\mu g_{\nu 0} + p_\nu g_{\mu 0}) + \frac{p\cdot q}{q^2}(p_\mu q_\nu - p_\nu q_\mu)\right]. \end{aligned} \tag{24}$$

This difference $S_{\mu\nu}$ is sometimes called the seagull term. For this difference to be a polynomial in q_0 we must have

$$\begin{aligned} T_2 - T_2^* &= 2D(p_0) + q^2 p_2(q_0),\\ T_1 - T_1^* &= -2C(p_0) + q^2 p_1(q_0) + (p\cdot q)^2 p_2(q_0), \end{aligned} \tag{25}$$

where p_1, p_2 are arbitrary polynomials in q_0 which can depend on p_n and $\vec{q}$. Thus,

$$\begin{aligned} T^*_{\mu\nu} = T_{\mu\nu} &+ 2C(p_0)(g_{\mu\nu} - g_{\mu 0}g_{\nu 0})\\ &- 2D(p_0)(p_\mu - p_0 g_{\mu 0})(p_\nu - p_0 g_{\nu 0})\\ &+ p_1(q_0)(q^2 g_{\mu\nu} - q_\mu q_\nu)\\ &+ p_2(q_0)[(p\cdot q)^2 g_{\mu\nu} + q^2 p_\mu p_\nu - (p\cdot q)(p_\mu q_\nu + p_\nu q_\mu)]. \end{aligned} \tag{26}$$

In order for $T^*_{\mu\nu}$ to be covariant, we must have $T^*_{\mu\nu}$ independent of timelike n with $n^2 = 1$. From this it is possible to show that p_1 and p_2 do not depend on n_λ. They are therefore covariant functions of q^2 and $p\cdot q$. Thus it is possible to define a "minimal" covariant and gauge-invariant $T^*_{\mu\nu}$ by dropping p_1 and p_2. This $T^*_{\mu\nu}$ is given by

$$\begin{aligned} T^*_{\mu\nu} = T_{\mu\nu} &+ 2C(p_0)(g_{\mu\nu} - g_{\mu 0}g_{\nu 0})\\ &- 2D(p_0)(p_\mu - p_0 g_{\mu 0})(p_\nu - p_0 g_{\nu 0}), \end{aligned} \tag{27}$$

which establishes the lemma.

Armed with these lemmas it is easy to prove our theorem. We recall that the expression for the self-mass of a hadron is given by

$$\delta M \propto \int \frac{d^4q}{(2\pi)^4}\frac{g_{\mu\nu}}{q^2}\tilde{T}^*_{\mu\nu}(\nu, q^2). \tag{28}$$

Here $\tilde{T}^*_{\mu\nu}$ differs from our $T^*_{\mu\nu}$ by at most a gauge-invariant and covariant term with no absorptive part. Let us assume that no such extra seagull term is needed. In order to get the Cottingham formula the integration contour has to be rotated in the q_0 plane (the so-called "Wick" rotation). The contribution of $T^*_{\mu\nu}$ along the semicircle at ∞, in the rest frame of the proton, is proportional to

$$\int_\infty^{i\infty} d\nu\, g_{\mu\nu}\frac{T^*_{\mu\nu}(\vec{q},\nu)}{\nu^2 - |\vec{q}|^2 + i\epsilon}. \tag{29}$$

We note here that $|\nu| \to \infty$ and the integral vanishes as long as $T^*_{\mu\nu}(\vec{q},\nu)$ grows less than linearly in ν for fixed $|\vec{q}|$. We will now show that this is the case for the $T^*_{\mu\nu}$ defined in Lemma 3. $T^*_{\mu\nu}$ can be written as $T^*_{\mu\nu} = T_{\mu\nu} + S_{\mu\nu}$, where $S_{\mu\nu}$ is the seagull term defined earlier. By definition $S_{\mu\nu}$ contains all polynomials in $q_0 = \nu$ (in the rest frame of the proton) so that the condition for Wick rotation is automatically satisfied for $T_{\mu\nu}$. Moreover $S_{\mu\nu}$ itself, for the class of currents for which $h_{00}(x,p)\delta(x_0) = 0$, is independent of q_0 and hence also causes no problems with regard to Wick rotation – which establishes the theorem.

Let us now examine the assumption $h_{00}(x,p)\delta(x_0) = 0$. We start by noting that $h_{00}(x,\vec{p}=0,p_0)\delta(x_0) = 0$ follows just from invariance under parity and translations and hence imposes no restrictions whatsoever on the forms of $h_{0i}(x,p)\delta(x_0)$ and $h_{ij}(x,p)\delta(x_0)$. Thus the condition $h_{00}(x,p)\delta(x_0) = 0$ for all Lorentz frames is clearly a strong assumption. It is, however, an assumption which is quite reasonable and is, in fact, true in low-order perturbation theory and in forms written down for $h_{\mu\nu}(x,p)$ consistent with scale invariance in inelastic e-p scattering.[11, 12] The condition $h_{00}(x,p)\delta(x_0) = 0$ does not, however, have to be satisfied by any $h_{\mu\nu}(x,p)$ which one can construct, as the following example illustrates. We start by listing general symmetry and other requirements which any $h_{\mu\nu}(x,p)$ is expected to satisfy:

(i) Causality requires $h_{\mu\nu}(x,p)=0$, for $x^2<0$.

(ii) Translation invariance requires $h_{\mu\nu}(x,p) = -h_{\nu\mu}(-x,p)$.

(iii) Hermiticity of $j_\mu(x)$ requires $h_{\mu\nu}(x,p) = -h^*_{\mu\nu}(x,p)$.

(iv) Parity gives $h_{\mu\nu}(x,p)=(-)^{g_{\mu 0}+g_{\nu 0}}h_{\mu\nu}(x_0,-\vec{x};p_0,-\vec{p})$.

(v) Time reversal times parity gives $h_{\mu\nu}(x,p) = h^*_{\mu\nu}(-x,p)$.

(vi) Gauge invariance gives $\partial_\mu h_{\mu\nu}(x,p)=0$.

Charge conjugation relates $h_{\mu\nu}(x,p)$ to a different process unless $|p\rangle$ is self-conjugate in which case charge conjugation is automatically satisfied here since we have two electromagnetic currents. It is easy to check that $h_{\mu\nu}(x,p)$ defined by

$$h_{\mu\nu}(x,p)=i(\Box g_{\mu\nu}-\partial_\mu\partial_\nu)(p\cdot\partial)^2\Delta(x,m^2), \qquad (30)$$

where

$$\Delta(x,m^2)=i\int\frac{d^4q}{(2\pi)^3}e^{iq\cdot x}(q_0)\delta(q^2-m^2)$$

satisfies all of these conditions and has

$$h_{00}(x,p)\delta(x_0)=-2ip_0\vec{\nabla}^2(\vec{p}\cdot\vec{\partial})\delta^4(x).$$

We would like to conclude by making a few remarks:

1. We have shown that it is possible to construct a covariant and gauge-invariant $T^*_{\mu\nu}(\nu,q^2)$ when $\delta(x_0)h_{00}(x,p)=0$ for all Lorentz frames so that the Cottingham formula is valid. We would like to suggest that the $T^*_{\mu\nu}$ constructed in Lemma 3 is the physical $\tilde{T}^*_{\mu\nu}$ which goes into the Cottingham formula. The reason for that is the following: $T^*_{\mu\nu}$ differs from $\tilde{T}^*_{\mu\nu}$ by gauge-invariant and covariant terms which do not contribute to the absorptive part. Such terms arise, in the language of field theory, from contact terms in the Lagrangian. It seems reasonable to drop such terms unless one is forced to keep them to satisfy some physical requirement.

2. With some reservations, we suspect that if higher derivatives are present in $h_{\mu\nu}(x,p)\delta(n\cdot x)$ then the Wick rotation is not allowed. This can be made plausible by the following argument. From the higher-derivative terms in $h_{\mu\nu}(x,p)\delta(n\cdot x)$ we would expect to get polynomials in $|\vec{q}|$ in the expression for $T_{\mu\nu}$. In order to make $T^*_{\mu\nu}$ covariant one would then have to add on polynomials in q_0 which would then spoil the condition for the Wick rotation to be valid. This argument is wrong for higher-derivative terms in $h_{\mu\nu}(x,p)\delta(n\cdot x)$ which disappear in the rest frame of the hadron.[13]

3. The condition for Wick rotations to be valid does not necessarily imply that δM is finite. In fact if $C(m)\neq 0$, δM could even be a quadratically divergent quantity. If δM is divergent, the order of integration can affect the validity of the Wick rotation. Our theorem applies only if the q_0 integration is done first.[14] Because of this, our result is not in contradiction to the work of Rabl,[5] who claims divergent δM is equivalent to not being able to perform the Wick rotation. He gets this result by doing the spatial integrals first.

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[1]W. N. Cottingham, Ann. Phys. (N.Y.) 25, 424 (1963).

[2]The variable ν is defined as $\nu=p\cdot q$, where p is the four-momentum of the hadron while q is the four-momentum of the photon.

[3]See, for example, D. Gross and H. Pagels, Phys. Rev. 172, 1381 (1968); R. Chanda, *ibid.* 188, 1988 (1969); M. Elitzur and H. Harari, Ann. Phys. (N.Y.) 56, 81 (1970); F. Buccella, M. Cini, M. DeMaria, and B. Tirozzi, Nuovo Cimento 64, 927 (1969).

[4]H. Pagels, Phys. Rev. 185, 1990 (1969); R. Jackiw, R. Van Royen, and G. West, Phys. Rev. D 2, 2473 (1970).

[5]A very clear discussion of the problems associated with the Cottingham formula is given in M. Elitzur and H. Harari, Ann. Phys. (N.Y.) 56, 81 (1970), although we find it hard to follow the comments made in the paper regarding Schwinger terms and the rotation of the contour. A. Rabl, Phys. Rev. 176, 2034 (1968), discusses the conditions under which Wick rotation is permitted. We feel Rabl has made very strong assumptions regarding Regge behavior in the entire ν-q^2 plane and the analyticity of $T^*_{\mu\nu}(\nu,q^2)$ for q^2 timelike. For a general discussion of divergence problems in electromagnetic mass shift calculations see, for example, D. Boulware and S. Deser, Phys. Rev. 175, 1912 (1968), and J. Cornwall and R. Norton, *ibid.* 173, 1637 (1968). The paper by Boulware and Deser also contains remarks on the Cottingham formula.

[6]By well defined we mean defined as a distribution with respect to the space of all infinitely differentiable test functions of bounded support. Also, when we write $[J_\mu(x),J_\nu(0)]$ we mean $[J_\mu(x),J_\nu(0)]-\langle 0|[J_\mu(x),J_\nu(0)]|0\rangle$ so that only Q numbers (Schwinger terms) appear in our discussions.

[7]V. Gupta and G. Rajasekaran, Phys. Rev. D 3, 677 (1971).

[8]M. Creutz and S. Sen, University of Maryland Report No. 71-073, 1971 (unpublished).

[9]The problem of "covariantizing" an amplitude has been discussed by a number of authors. See, for example, L. S. Brown, Phys. Rev. 150, 1338 (1966); L. S. Brown and D. Boulware, *ibid.* 156, 1724 (1967); R. F. Dashen and S. Y. Lee, *ibid.* 187, 2017 (1969); D. J. Gross and R. Jackiw, Nucl. Phys. B14, 269 (1969). Instead of using any of the general results, we preferred to covariantize $T_{\mu\nu}$ in this straightforward way.

[10]J. Bjorken, Phys. Rev. 148, 1467 (1966).

[11]K. Johnson and F. E. Low, Progr. Theoret. Phys. (Kyoto) Suppl. 74, 37 (1966), find that $[V_0(x), V_0(0)]\delta(x_0) = 0$ in their model.

[12]For example, in R. Jackiw, R. Van Royen, and G. B. West, Phys. Rev. D 2, 2473 (1970).

[13]Our example of an $h_{\mu\nu}(x,p)$ which has $\delta(x_0)\, h_{00}(x,p) \neq 0$ leads to a $T^*_{\mu\nu}$ which cannot be Wick rotated. However, if $h_{\mu\nu}(x,p)$ is modified to

$$\tilde{h}_{\mu\nu}(x,p) = i\,(g_{\mu\nu}\Box - \partial_\mu \partial_\nu)[(p\cdot\partial)^2 - m^2\Box]\Delta(x,m),$$

then the expression for $h_{00}(x,p)\delta(x_0)$ remains unmodified but the $T^*_{\mu\nu}$ obtained from it *can be Wick rotated.* This example clearly indicates that the condition that $h_{00}(x,p) \times \delta(x_0)$ vanish in all frames is a sufficient but not a necessary condition for Wick rotation of the contour to be permitted.

[14]Doing the q_0 integration first is the "natural" thing to do as it leads to Feynman rules in a perturbative framework. We would like to thank J. Sucher for pointing this out to us.

PHYSICAL REVIEW D VOLUME 4, NUMBER 10 15 NOVEMBER 1971

Self-Consistent Pomeranchon Coupling Ratios in the Multiperipheral Model*

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Given the two leading eigenvalues and eigenfunctions of the resonance (low-subenergy) component of a multiperipheral kernel and assuming lower eigenvalues to be unimportant, it is shown how the mixture corresponding to the Pomeranchon eigenfunction may be calculated from considerations of self-consistency. The method is illustrated in a multiperipheral model with pseudoscalar-meson links by associating the two leading unperturbed eigenstates with the 2^+ particles $f(1260)$ and $f'(1514)$.

I. INTRODUCTION

A recently developed multiperipheral model describes the leading Regge singularity with vacuum quantum numbers, the so-called "Pomeranchon," as arising from a self-generating perturbation that splits an idealized "primitive" Regge-pole spectrum into fine structure.[1,2] The primitive spectrum belongs to a simplified multiperipheral integral equation whose kernel is truncated so as to lack the Pomeranchon-generated long-range correlations in longitudinal rapidity. When only short-range (resonance) correlations are included in the kernel, the generated Regge singularities consist entirely of well-spaced poles. Inclusion of the weak but long-range Pomeranchuk component of the kernel introduces a weak branch point in crossed-channel angular momentum, and in the associated multisheeted J Riemann surface, poles occur at slightly different positions on the different sheets. This combination of branch point with closely spaced poles, first identified by Frazer and Mehta,[3] is what we have called "fine structure."

It has been conjectured that the Pomeranchon (P) should be identified with a real pole on the first (physical) sheet of the J surface while the nearest pole on the second sheet, whose position is slightly complex, should be identified with the P'. The lower-lying P'' would similarly be located on the second sheet. If the separation between P and P' is small compared with the distance to other poles, it can be shown that the ratios of P and P' couplings should be almost equal – that is, the P and P' "vectors" should be almost "parallel." A rough parallelism between P and P' couplings is an established experimental fact, but the observed P and P' vectors appear not to be exactly parallel.[4] For example, the P' is often associated with the 2^+ $f(1260)$ and described (e.g., in the quark model) by a vector whose $K\bar{K}$ component has half the amplitude of the $\pi\pi$ component, while the observed P seems closer to being an SU_3 singlet, with equal $\pi\pi$ and $K\bar{K}$ components. Recently it has been suggested by Carlitz, Green, and Zee[5] (CGZ) that the P vector should have a significant component parallel to the $f'(1514)$ as well as a component parallel to $f(1270)$. The special model employed by CGZ, however, is inconsistent in its treatment of the Pomeranchon vector in that these authors begin by assuming the Pomeranchon to be a pure SU_3 singlet while their finally calculated vector has a substantial octet component. The present paper explores the general CGZ suggestion within a multiperipheral framework and shows how self-consistency can be achieved for the Pomeranchon coupling ratios.

II. MIXING OF THE UNPERTURBED EIGENSTATES

We employ the notation of Sec. V of Ref. 2 where