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<sup>1</sup>S. D. Drell and T.-M. Yan, *Phys. Rev. Letters* **24**, 855 (1970).

<sup>2</sup>R. P. Feynman, *Photon Hadron Interactions* (Benjamin, New York, to be published), and talk presented at the "Neutrino '72" Conference, Balatonfüred, Hungary, 1972 (unpublished). Feynman has suggested the quantum numbers of the parton are measured by the average quantum numbers found in the parton fragmentation region.

<sup>3</sup>S. Berman, J. D. Bjorken, and J. Kogut, *Phys. Rev. D* **4**, 3388 (1971).

<sup>4</sup>M. Gronau, F. Ravndal, and Y. Zarmi, Caltech Report No. CALT-68-367 (unpublished).

<sup>5</sup>For the distinction between the parton and photon fragmentation regions, see J. D. Bjorken, *Phys. Rev. D* **7**, 282 (1973).

D **7**, 282 (1973).

<sup>6</sup>Using the variables  $u_1$  and  $P_{\perp}^2$  which are finite in both parton and proton fragmentation regions, Ref. 4 defines structure functions which scale with the same powers of  $\nu$  as those of purely inclusive electroproduction. This scaling behavior is equivalent to Eqs. (5) and (6). (The correct scaling behavior is also implicitly contained in Refs. 2 and 3.)

<sup>7</sup>These scaling relations can also be derived from naive dimensional arguments.

<sup>8</sup>C. J. Bebek, C. N. Brown, C. A. Lichtenstein, M. Herzlinger, F. M. Pipkin, K. Sistrerson, D. Andrews, K. Berkelman, D. C. Cassel, D. L. Hartill, and N. Hicks, report submitted to the Sixteenth International Conference on High Energy Physics, Batavia, Ill., 1972 (unpublished).

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## Comment on "Are There Fixed Singularities in $T_1$ ?"\*

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We show that the conclusion of Zee on the presence of fixed  $J=0$  singularities in virtual Compton scattering is unfounded. Thus, there is no model-independent argument for such a behavior.

In a recent article Zee<sup>1</sup> has attempted to show on the basis of some commonly accepted assumptions that the amplitude  $T_1$  of virtual forward Compton scattering must contain in its Regge expansion an asymptotically constant term. Such a possible term has been much discussed in the literature; indeed, there is weak experimental evidence for its existence.<sup>2</sup> In this note we show that Zee has assumed his result, and in fact nothing can be said about such a term.

In order to establish notation and pinpoint the crucial assumption, we quote the relevant passages from Ref. 1:

Our assumptions are the following:

- (a) The Bjorken-Johnson-Low limit  $\lim_{\text{BjL}} Q_0^2 T_{ij}$  exists.
- (b) The behavior of the current commutator

near the light cone is relevant for the deep-inelastic region.

Assumption (b) actually implies nonforward scaling.

We now demonstrate our assertion. The target spin-averaged nonforward Compton amplitude may be decomposed as

$$T_{\mu\nu}(q, q', p, p') = -g_{\mu\nu} T_1(\nu, Q^2, t, \delta) + \frac{P_\mu P_\nu}{m^2} T_2(\nu, Q^2, t, \delta) + \dots, \quad (1)$$

where

$$P = \frac{1}{2}(p + p'), \quad Q = \frac{1}{2}(q + q'), \quad \Delta = p - p' = q' - q, \\ \nu = P \cdot Q, \quad \delta = Q \cdot \Delta = q'^2 - q^2, \quad t = \Delta^2,$$

and

$$\omega = -Q^2/2\nu.$$

Suppose that there is *no* fixed singularity in the  $J$  plane. Then  $T_1$  presumably satisfies an unsubtracted dispersion relation for  $t < t_0$  for some  $t_0 < 0$ , viz.,

$$T_1(\nu, Q^2, t, \delta) = \int_{\nu_0^2}^{\infty} \frac{d\nu'^2}{\nu'^2 - \nu^2} W_1(\nu', Q^2, t, \delta) \quad (2a)$$

$$= 2\omega^2 \int \frac{d\omega'}{\omega'} \frac{W_1(\omega', Q^2, t, \delta)}{\omega^2 - \omega'^2}. \quad (2b)$$

We now let  $Q_0 \rightarrow i\infty$  keeping  $t$  and  $\delta$  fixed:

$$T_1(\nu, Q^2, t, \delta) \underset{\text{BJL}}{\sim} 2 \int_0^1 \frac{d\omega'}{\omega'} \lim_{Q^2 \rightarrow -\infty} W_1(\omega', Q^2, t, \delta). \quad (3)$$

Assumption (a) then implies that

$$\lim_{Q^2 \rightarrow -\infty} Q^2 W_1(\omega, Q^2, t, \delta) = -\bar{F}_1(\omega, t, \delta). \quad (4)$$

[We exclude the possibility that  $\lim_{Q^2 \rightarrow -\infty} W_1(\omega, Q^2, t, \delta) = F_1(\omega, t)$  and that  $\int_0^1 d\omega \omega^{-1} F_1(\omega, t) = 0$  for all  $t < t_0$ .]

From Eq. (4), Zee goes on to demonstrate his assertion.

However, to go from Eq. (3) to Eq. (4) requires exclusion of the behavior mentioned in parentheses after Eq. (4). *But this is the expected behavior when there is no fixed Regge singularity.* In other words, at this point the result has been assumed.

One might immediately object that it is unlikely for the parenthetical behavior to occur for all  $t$  such that the integral converges. The remainder of this note will show that this is not the case. First we will give a simple example where this behavior occurs; then, we will show that a slight extension of Zee's argument leads to an absurdity.

Our example is

$$T_1(\nu, Q^2, t, \delta) = -\left\{\frac{1}{2}[1 + (1 - \omega^{-2})^{1/2}]\right\} \alpha_1(t) + \left\{\frac{1}{2}[1 + (1 - \omega^{-2})^{1/2}]\right\} \alpha_2(t),$$

where  $\alpha_1(t)$  and  $\alpha_2(t)$  are two normal moving Regge trajectories. The complicated square-root structure is introduced to give cut  $\nu$ -plane analyticity. We take the branch of the square root with positive real part. By selecting  $\alpha_1(0) = 1$  and  $\alpha_2(0) \approx \frac{1}{2}$  corresponding to the Pomeranchukon and some lower trajectory such as the  $P'$ , we satisfy the positivity condition

$$\text{Im}T_1(\nu, Q^2 < 0, t = 0, \delta) \geq 0.$$

This amplitude clearly scales and has normal Regge behavior. In fact, under Fourier transformation this amplitude gives a local commutator. In the Bjorken-Johnson-Low limit, the amplitude behaves in the assumed  $Q_0^{-2}$  manner. Thus this  $T_1$  satisfies assumptions (a) and (b) of Ref. 1; however, it has no fixed pole.

One might feel that in this example there is a rather unlikely cancellation between the two Regge terms to give the proper BJL limit. In order to argue against this point of view, we now show that Zee's reasoning can give an absurd result. Motivated by continuous-moment sum rules, we consider the function

$$\bar{T}_1(\nu, Q^2, t, \delta) = [1 + (1 - \omega^{-2})^{1/2}]^{-\lambda} T_1(\nu, Q^2, t, \delta),$$

where  $0 < \lambda < 1$ . One can readily see that  $\bar{T}_1$  also satisfies the assumptions on  $T_1$  which Zee uses in his argument for a fixed pole. Thus, by this reasoning,  $\bar{T}_1$  should have a constant piece in its asymptotic behavior as  $\nu$  goes to infinity at fixed  $Q^2$ ,  $t$ , and  $\delta$ . This corresponds to a fixed behavior in  $T_1$  of the form  $\nu^\lambda$ . But  $\lambda$  is arbitrary. Therefore  $T_1$  must have a fixed singularity at every value of  $J$  in the range  $0 < J < 1$ . This is clearly absurd.

We have shown that the argument of Ref. 1 for a fixed  $J=0$  pole in the virtual Compton amplitude assumes its conclusion. Thus there exists no general theoretical argument, independent of specific models, for such a singularity.

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<sup>1</sup>A. Zee, Phys. Rev. D 5, 2829 (1972), and references

therein.

<sup>2</sup>M. J. Creutz, S. D. Drell, and E. A. Paschos, Phys. Rev. 178, 2300 (1969); M. Damashek and F. J. Gilman, Phys. Rev. D 1, 1319 (1970); C. A. Dominguez, C. Ferro Fontan, and R. Suaya, Phys. Letters 31B, 365 (1970); F. Close and J. Gunion, Phys. Rev. D 4, 742 (1971).