er to the zero of t' and the data analysis will have accompanying uncertainties.

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Confinement and the Critical Dimensionality of Space-Time

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Using Monte Carlo techniques, we study pure SU(2) gauge fields in four and five space-time dimensions and a compact SO(2) gauge field in four dimensions. Ultraviolet divergences are regulated with Wilson's lattice prescription. Both SU(2) in five dimensions and SO(2) in four dimensions show clear phase transitions between the confining regime at strong coupling and a spin-wave phase at weak coupling. No phase change is seen for the four-dimensional SU(2) theory.

The standard theory of hadronic interactions is based on quarks interacting with non-Abelian gauge fields. The viability of this picture depends on the conjectured phenomenon of confinement, wherein the only physically observable particles are invariant under the gauge group. Thus far, the only demonstration of this property is in the strong-coupling limit and with a space-time lattice regulating ultraviolet divergences.[1] Approximate renormalization-group arguments[2] suggest that four space-time dimensions represent a criical case where confinement persists for all couplings when the gauge group is non-Abelian. In contrast, Abelian groups should exhibit a phase transition to a nonconfining weak-coupling phase containing massless gauge bosons. Thus arises the conjecture that in our four-dimensional (4D) world, the lattice formulation of electrodynamics can avoid confinement of electrons, while the continuum limit of the strong-interaction gauge theory can exhibit asymptotic freedom, a vanishing coupling at short distances.

Recent Monte Carlo results have given mixed support for these arguments. For the four-dimensional gauge-invariant Ising model, the observed transition is first order, contrary to the approximate renormalization-group prediction of a second-order transition analogous to that in the conventional two-dimensional Ising model.[3] However, for Z_n with $n \geqslant 5$ and SO(2) symmetries, the predicted similarities between the four-dimensional gauge models and the two-dimensional spin systems are confirmed.[4]

In this note I report results on Monte Carlo studies of SU(2) gauge theory. To show the critical nature of four dimensions I ran these simulations for both four- and five-dimensional lattices. I also make comparisons with the Abelian group SO(2) [isomorphic to U(1)]. I work with pure gauge fields on the assumption that the addition of a few fermion species represents a perturbation that will not spoil confinement. Although the group of physical interest is SU(3), I study SU(2) because of its simpler structure. As confinement is connected with disorder in the lattice formulation, and as adding more degrees of freedom should increase disorder, confinement with SU(2) gauge fields should imply confinement with SU(3).

The system is formulated on a hypercubical lattice. Associated with the link joining any pair of nearest-neighbor sites i and j is an element U_{ij} of the gauge group (i and j label sites and should not be confused with the implicit matrix indices on the group elements). The wave function of a particle traversing the respective link undergoes an internal-symmetry rotation corresponding to U_{ij}. The reverse path gives the conjugate rotation

$$U_{ji}=(U_{ij})^{-1}, \tag{1}$$

where the inverse is in the group sense. The quantum theory is defined via the path integral

$$Z=\int\Big(\prod_{\{i,j\}} dU_{ij}\Big)e^{-\beta S(U)}, \tag{2}$$

where the integral includes all links and uses the invariant group measure. The action is that defined by Wilson,

$$S(U)=\sum_{\Box}S_{\Box}, \tag{3}$$

where the sum extends over all elementary squares or "plaquettes" $\Box$ and

$$S_{\Box}=1-\tfrac{1}{2}\,\mathrm{Tr}(U_{ij}U_{jk}U_{kl}U_{li}). \tag{4}$$

Here i, j, k, and l are some labeling of the sites going around the square $\Box$. The normalization is such that for the groups SU(2) and SO(2) any plaquette contributes a number between zero and two to the action. As shown by Wilson,[1] this action reduces in the classical continuum limit to the usual gauge-theory action with β proportional to the inverse square of the coupling constant.

Equation (1) is formally identical to the partition function of a statistical mechanical system with Hamiltonian S and at inverse temperature β. The Monte Carlo algorithm consists of successively touching a heat bath to each link of the lattice while holding fixed the group elements on the remaining links. Repeating this procedure will eventually produce a sequence of states which simulates an ensemble of such systems in thermal equilibrium.[5] Green's functions for the quantum theory follow from correlation functions in the states of the ensemble.

Beginning in some initial configuration, we pass through the entire lattice varying one link at a time. At each link's turn, a new group element g is selected to occupy that position. This choice is made randomly from the entire gauge group with weighting proportional to the Boltz-

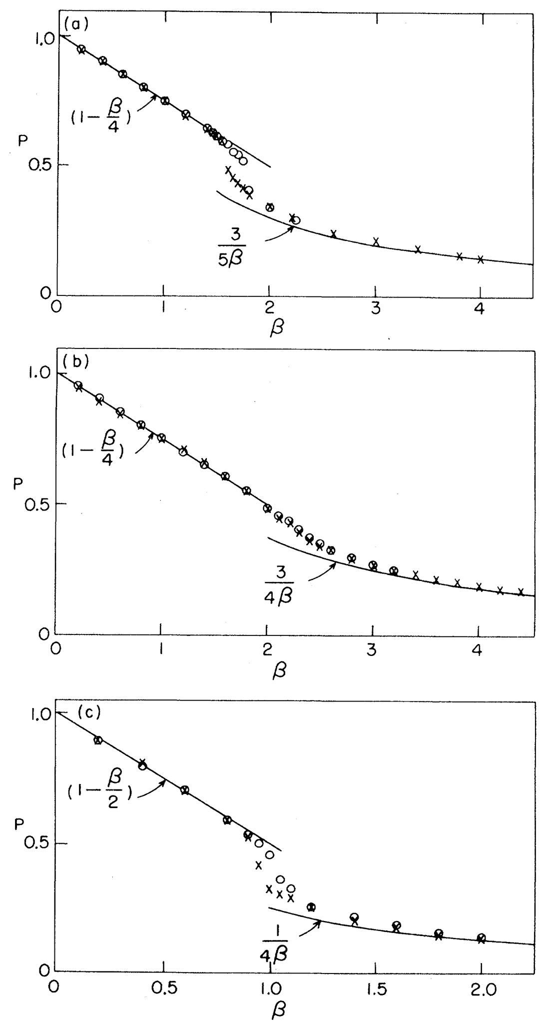

FIG. 1. The average plaquette as a function of β as obtained on cooling and heating the gauge systems with (a) SU(2) in five dimensions, (b) SU(2) in four dimensions, and (c) SU(2) in four dimensions. Crosses, heating; circles, cooling.

mann factor

$$B(g) = \exp[-\beta S(g)], \quad (5)$$

where $S(g)$ is the action evaluated with the given link having group element g and all other links fixed with their previous values. The old value for the current link plays no direct role in this procedure. In what follows an iteration is defined as one application of this algorithm to each link in the lattice.

For the four-dimensional models we use a $5\times5\times5\times5$ lattice while for the five-dimensional simulation we work on a $4\times4\times4\times4\times4$ lattice. To minimize surface effects we impose periodic boundary conditions. As an order parameter we use the average action per plaquette,

$$P = \langle S_{\square} \rangle. \quad (6)$$

This quantity is proportional to the internal energy of the statistical system and runs between zero and one as β decreases from infinity to zero.

In Fig. 1 I show the results of a thermal cycle on the models. Each point is obtained from either higher or lower temperature by iterating the Monte Carlo procedure until no net trend in P is observed over six iterations. Plotted is the average value of P over these six iterations. The heating runs are initiated with a totally ordered lattice while the cooling runs start with all link variables chosen randomly. Note that the four-dimensional SO(2) and the five-dimensional SU(2) models show clear hysteresis effects indicative of phase transitions. The four-dimensional SU(2) model shows no similar gross structure, although convergence appears to be slightly reduced in the region $2.2 \lesssim \beta \lesssim 2.5$. I further discuss this region below.

In order to quote critical temperatures for the observed transitions, I select a value P_0 for P in the middle of the hysteresis loops and then iterate while adjusting β until P fluctuates around P_0. Choosing $P_0 = 0.52$ for SU(2) in five dimensions and $P_0 = 0.4$ for SO(2) in four dimensions, I obtain

$$\beta_c = 0.987 \pm 0.023, \quad \text{SO(2) in 4D}, \quad (7)$$

$$\beta_c = 1.642 \pm 0.015, \quad \text{SU(2) in 5D}. \quad (8)$$

To investigate whether the transitions are of first or higher order, I made extended runs at

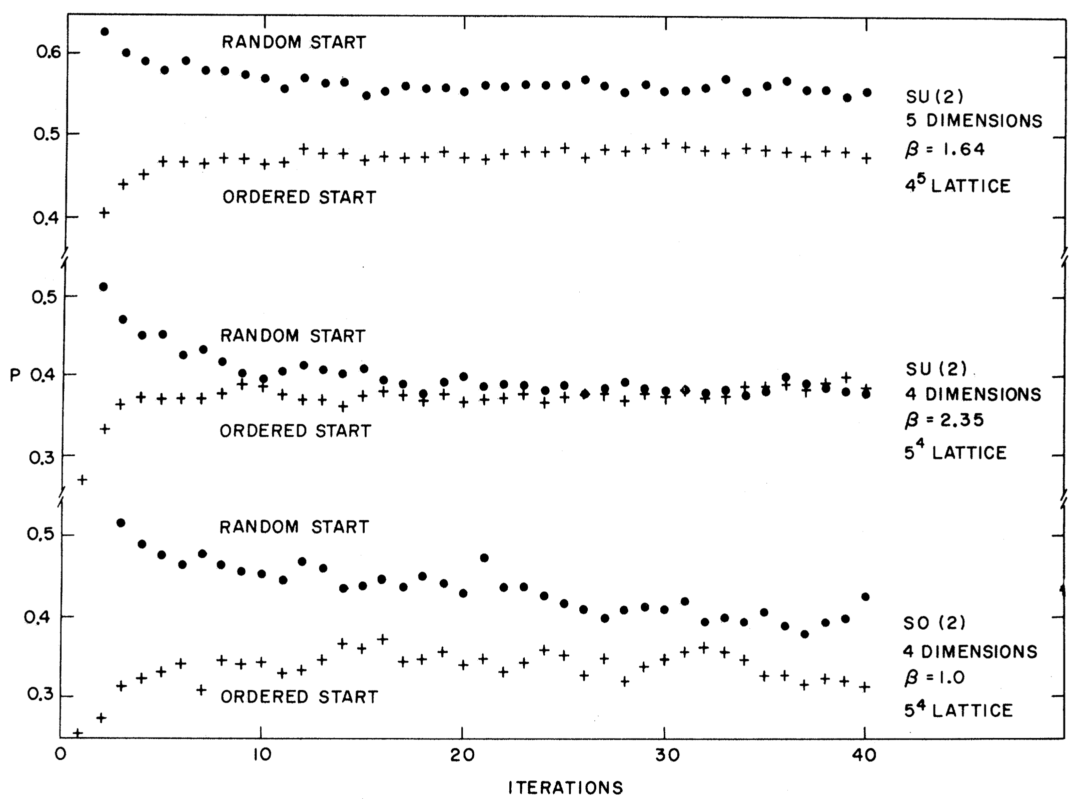

FIG. 2. The average plaquette as a function of number of iterations at a fixed β.

the critical temperatures with both ordered and disordered starts. In Fig. 2 we see that for SU(2) in five dimensions the two runs stabilize at different values, indicative of a first-order transition. In contrast, for SO(2) in four dimensions these runs show large fluctuations and continue to converge slowly, suggesting a higher-order transition.

In Fig. 2 I also show the results of similar runs with the four-dimensional SU(2) model at $\beta = 2.35$. This corresponds to a temperature in the middle of the slow-convergence region alluded to above. The two runs converge after about thirty iterations, while fluctuations are considerably controlled relative to those seen in the SO(2) model. I feel that the reduced convergence in this region is not evidence for a phase transition, but rather a consequence of the critical nature of four dimensions.

At low β (high temperature) the points follow the strong-coupling limit

$$P = 1 - \tfrac{1}{2}\beta + O(\beta^3) \quad \text{for SO(2)}, \tag{9}$$

$$P = 1 - \tfrac{1}{4}\beta + O(\beta^3) \quad \text{for SU(2)}. \tag{10}$$

The large-β behavior can be estimated by keeping only those terms in the action which are quadratic in parameters describing the group manifold. This yields a Gaussian path integral and implies

$$P \xrightarrow[\beta \to \infty]{} n/\beta d, \tag{11}$$

where n is the number of group generators and d is the dimensionality of space-time. The functions in Eqs. (9)–(11) are plotted along with the "data" in Fig. 1. Note that this inverse-β behavior at large β is approached in all the models. I do not expect any further phase transitions for β above the onset of this spin-wave behavior.

In conclusion, I have presented Monte Carlo evidence that the confinement phase of SU(2) lattice gauge theory in four dimensions extends to all values of coupling. This means that the continuum limit of this theory simultaneously exhibits confinement and asymptotic freedom. Of course this is not an analytic proof; indeed, there could exist a subtle transition not readily observable in the average plaquette. I regard this as unlikely in the light of the extreme clarity of the transitions seen with the other models.

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Limits on *CP*-Invariance Violation in $K_{\mu 3}$ Decays

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Measurements of the polarization of μ^+ from the decay of K_L^0 mesons gives a mean value of 0.0021 ± 0.0048 for the polarization in the direction $(\vec{p}_\pi \times \vec{p}_\mu)$ normal to the plane of decay. The ratio of the normal to the transverse polarization in the c.m. system is 0.0041 ± 0.0092 and the value of Imξ is deduced to be 0.012 ± 0.026, not significantly different from Im$\xi = 0.008$ expected if the decay were invariant under CP (or T).

The observed breakdown[1] in CP or T invariance, noted in K_L decays, can be described by either of two sets of theoretical conjectures. It is possible that a moderately weak (milliweak) CP-nonconserving force acts in second order or that a much weaker (superweak) CP-nonconserv-