Numerical studies of Wilson loops in SU(3) gauge theory in four dimensions

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(Received 11 June 1982)

Monte Carlo simulations are used to calculate Wilson loops for pure SU(3) gauge theory on a 6^4 lattice. Previous measurements of the scale parameter Λ_0 are improved.

The gauge group SU(3) has been examined in several recent Monte Carlo studies.[1–3] In Ref. 1 for instance most of the data were generated on 4^4 lattices with one data point generated on a 6^4 lattice. Since SU(3) is the gauge group of quantum chromodynamics (QCD), it is reasonable to improve our data sample and hence make a more accurate determination of the Λ_0 scale parameter. In the present paper, we report Monte Carlo simulations on a 6^4 lattice at 57 values of the inverse temperature and determine all Wilson loops up to size 3×3.

We work in a hypercubical lattice in four Euclidean dimensions.[4,5] On the link $\{ij\}$ joining nearest-neighbor lattice sites signified by i and j sits an $N\times N$ unitary-unimodular matrix U_{ij} of the group SU(N), with the condition that

$$U_{ji}=(U_{ij})^{-1} \ .$$

We define our partition function by

$$Z(\beta)=\int\left(\prod_{\{i,j\}} dU_{ij}\right)\exp(-\beta S[U]) \ ,$$

where β is the inverse temperature given by $\beta=2N/g_0{}^2$ with g_0 the bare coupling constant. The measure in the above integral is the SU(N) normalized invariant Haar measure. The action S is defined as the sum over all unoriented plaquettes $\Box$ such that

$$S[U]=\sum_{\Box} S_{\Box}=\sum_{\Box}\left(1-\frac{1}{N}\mathrm{Re}\,\mathrm{Tr}\,U_{\Box}\right) \ .$$

Here $U_{\Box}$ is the parallel transporter around a plaquette. Periodic boundary conditions were used throughout our calculations and the lattice was put in equilibrium by the method of Metropolis *et al.*[6] From now on we specialize to $N=3$.

We define the rectangular Wilson loops[7] by the expectation value

$$W(I,J)=\tfrac{1}{3}\langle \mathrm{Re}\,\mathrm{Tr}\,U_C\rangle \ ,$$

where the I by J closed rectangular contour is denoted by C and U_C is the parallel transporter or product of link variables around C. The leading-order high-temperature expansion for the Wilson loop is

$$W(I,J)=(\beta/18)^{IJ} \ , \qquad (1)$$

while the leading-order low-temperature expansion

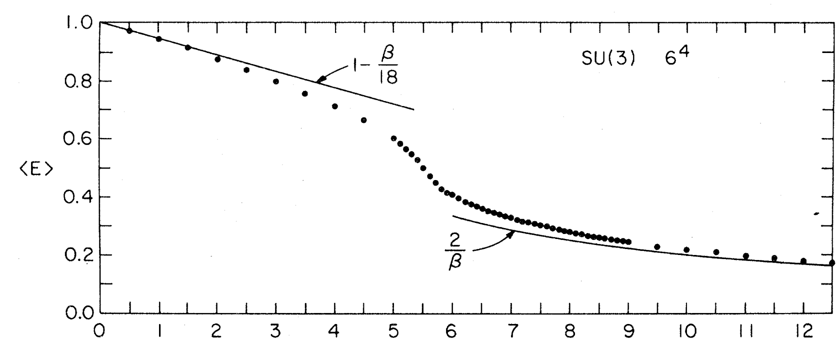

FIG. 1. The average action per plaquette $\langle E\rangle$ for pure SU(3) gauge theory on a 6^4 lattice as a function of the inverse temperature β. The curves represent the leading-order high- and low-temperature expansions of Eqs. (1) and (2), respectively.

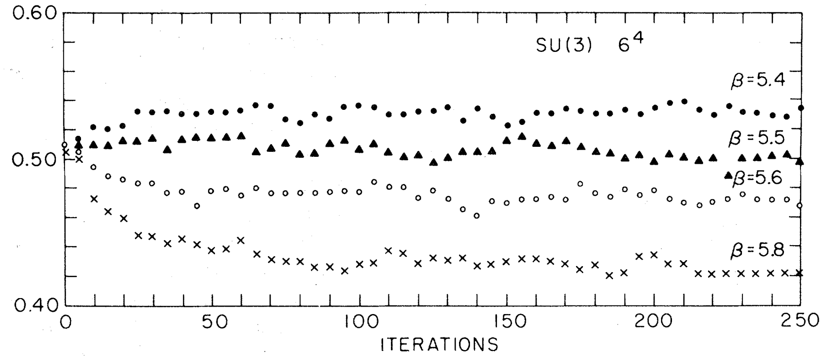

FIG. 2. The evolution of the average action per plaquette $\langle E\rangle$ for pure SU(3) gauge theory on a 6^4 lattice as a function of the number of iterations through the lattice for mixed-phase starting lattices for various values of the inverse temperature β.

for the average action per plaquette is

$$\langle E\rangle = 1 - W(1,1) = 2/\beta + O(\beta^{-2}) \ . \qquad (2)$$

For asymptotically large Wilson loops we expect

$$W \sim \exp(-A - K\times\text{area} - C\times\text{perimeter}) \ ,$$

where for a given β, A, K, and C are constants. When the asymptotic behavior applies, we extract the string tension K by evaluating the quantity

$$\chi(I,J) = -\ln\left[\frac{W(I,J)\,W(I-1,J-1)}{W(I,J-1)\,W(I-1,J)}\right] \ .$$

The leading-order high-temperature expansion for the string tension is given by

$$\chi(I,J) = -\ln(\beta/18) + O(\beta^2) \ . \qquad (3)$$

Asymptotic freedom determines how the lattice spacing varies with bare coupling for a continuum limit. This introduces a scale parameter Λ_0 defined by

$$\Lambda_0 = \lim_{a\to 0}\frac{1}{a}[\gamma_0 g_0^2(a)]^{(-\gamma_1/2\gamma_0^2)}\exp\left[\frac{1}{2\gamma_0 g_0^2(a)}\right] \ , \qquad (4)$$

where, for SU(3), we have

$$\gamma_0 = \frac{11}{16\pi^2} \quad \text{and} \quad \gamma_1 = \frac{51}{128\pi^4} \ ,$$

and a is the lattice spacing.

In Fig. 1 we show the average action per plaquette $\langle E\rangle$ as a function of the inverse temperature on a 6^4 lattice. In carrying out these calculations, we first performed 200 iterations through the 6^4 lattice with 20 Monte Carlo updates per link. This resulted in the space-time lattice being in equilibrium. We then averaged over the next 100 iterations through the lattice. We used disordered starting lattices for $\beta \leqq 5.5$, mixed-phase[5] starting lattices for $5.5 < \beta < 9.0$, and ordered starting lattices for $\beta > 9.0$. Our results in Fig. 1 agree well with the leading-order high- and low-temperature expansions of Eqs. (1) and (2),

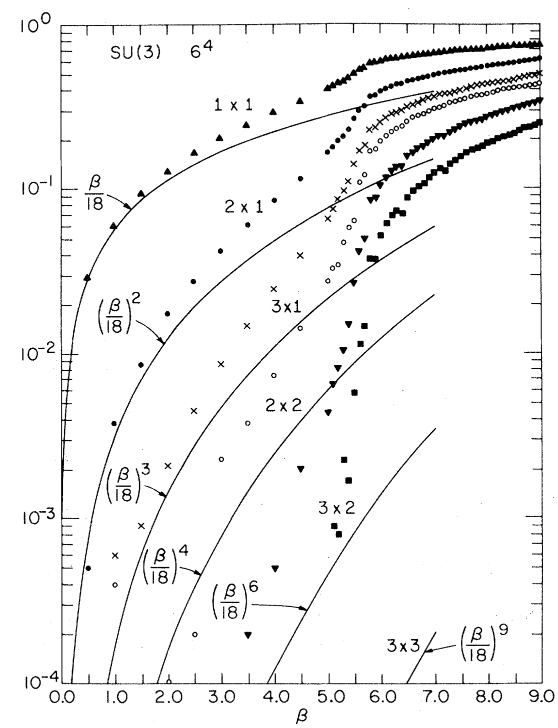

FIG. 3. The Wilson loops $W(I,J)$ for pure SU(3) gauge therory on a 6^4 lattice as a function of the inverse temperature β. The upward triangles represent $I=J=1$, the solid circles represent $I=2$, $J=1$, the crosses represent $I=J=2$, the downward triangles represent $I=3$, $J=2$, and the squares represent $I=J=3$. The curves represent the leading-order high-temperature expansion of Eq. (1).

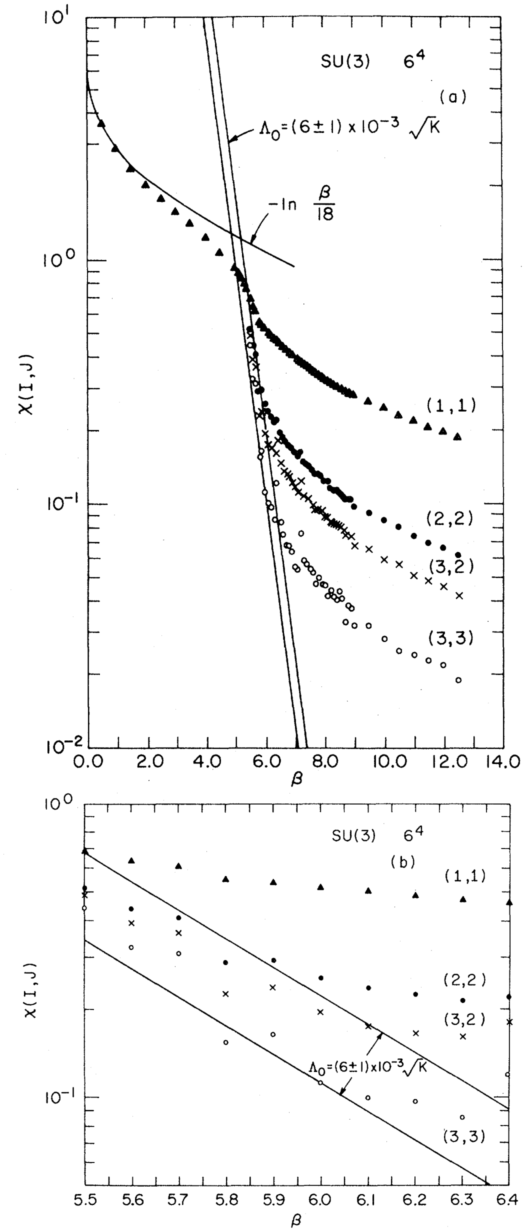

FIG. 4. The string tension $\chi(I,J)$ for pure SU(3) gauge theory on a 6^4 lattice as a function of the inverse temperature β. The triangles represent $I=J=1$, the solid circles represent $I=J=2$, the crosses represent $I=3$, $J=2$, and the open circles represent $I=J=3$. Also shown in the diagram is the leading-order high-temperature expansion of Eq. (3).

respectively. Figure 2 shows some of the mixed-phase runs for the average action per plaquette in the vicinity of the crossover between the high- and low-temperature regions. In Fig. 3 we show the Wilson loops up to size 3×3. The leading-order high-temperature expansions are also shown for comparison.

The logarithmic ratios $\chi(I,J)$ for $(I,J)=(1,1)$, (2,2), (3,2), and (3,3) are shown as a function of the inverse temperature β in Fig. 4(a). Our results agree with the leading-order high-temperature expansion of Eq. (3) up to $\beta\simeq1.0$. Obviously, higher-order terms are needed to bring about agreement with the Monte Carlo data in a larger range in β.

In the figure we show a band corresponding to the behavior of Eq. (4) with

$$\Lambda_0=(6\pm1)\times10^{-3}\sqrt{K}\ .$$

As in our previous analysis, the error is a subjective estimate. Putting in the Hasenfratz-Hasenfratz[8] factor relating Λ_0 to the parameter Λ^{MOM} characterizing the momentum-space three-point vertex in the Feynman gauge

$$\Lambda^{\mathrm{MOM}}/\Lambda_0=83.5\ ,$$

we obtain

$$\Lambda^{\mathrm{MOM}}=(0.5\pm0.1)\sqrt{K}\ .$$

This represents about 200 MeV if we use the Regge slope to estimate K.

We would like to thank the Science and Engineering Research Council of Great Britain for the award of research grants (Grants No. NG-0983.9 and No. NG-1068.2) to buy time on the CRAY-1S computer at Daresbury Laboratory where part of this calculation was carried out and the University of London Computer Centre for the granting of discretionary time on their CRAY-1S where the rest of this calculation was performed. The calculations of the present paper took the equivalent of 75 CDC 7600 hours to perform. This research was also carried out in part under the auspices of the U.S. Department of Energy under Contract No. DE-AC02-76CH00016.

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