BNL 33016

LATTICE GAUGE THEORIES

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April 1983

Talk presented at the Non-Perturbative Quantum Chromodynamics Conference, Oklahoma State University, March 7 - 9, 1983

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In the last few years lattice gauge theory has become the primary tool for the study of nonperturbative phenomena in gauge theories. The lattice serves as an ultraviolet cutoff, rendering the theory well defined and amenable to numerical and analytical work. Of course, as with any cutoff, at the end of a calculation one must consider the limit of vanishing lattice spacing in order to draw conclusions on the physical continuum limit theory. The lattice has the advantage over other regulators that it is not tied to the Feynman expansion. This opens the possibility of other approximation schemes than conventional perturbation theory. Thus Wilson used a high temperature expansion to demonstrate confinement in the strong coupling limit. Monte Carlo simulations have dominated the research in lattice gauge theory for the last four years, giving first principle calculations of nonperturbative parameters characterizing the continuum limit.

Before reviewing some of the recent results with lattice calculations, I wish to spend a few minutes reviewing the parameters of the theory of the strong interactions. First of all there are the quark masses. These presumably arise from some grand unification of the forces of nature, and are generally regarded as uncalculable in the gauge theory of the strong interactions alone. Their values are related to the pseudoscalar meson masses, which would vanish in a chirally symmetric world of vanishing quark masses. The remarkable feature of the strong interactions is that these are the only parameters. In principle all dimensionless quantities, such as the ratio of the ρ meson mass to the nucleon mass, are determined once the quark masses are given. This applies not only to mass ratios, but also to quantities such as the pion-nucleon coupling constant which once was considered a possible expansion parameter for a fundamental theory of baryons and mesons.

But what about the strong coupling constant; isn't that a parameter? Indeed, it is not; the coupling drops out of physical quantities via the phenomenon of dimensional transmutation [1], which I will now discuss in the context of the lattice regulator. On a lattice it is natural to consider a mass in units of the lattice spacing. In statistical mechanics this represents an inverse correlation length

$$\xi^{-1} = ma \quad , \tag{1}$$

where m is the mass in question and a the lattice spacing. Now for a continuum limit we wish to take the lattice spacing to zero while holding the mass m at its physical value. Eq. (1) immediately implies that in this limit the correlation length must diverge. This is the source of the statement that in statistical mechanics one must go to a critical point to obtain a continuum limit.

For an asymptotically free theory, as considered here, we know something about how the coupling varies for the continuum limit. As the bare coupling is an effective coupling at the scale of the cutoff, it should decrease logarithmically with the cutoff [2]

$$g_0^{\,2} = \left(\gamma_0 \ln(a^{-2}\Lambda_0^{-2}) + (\gamma_1/\gamma_0)\ \ln\ln(a^{-2}\Lambda_0^{-2}) + (g_0^{\,2})\right)^{-1} \quad . \tag{2}$$

Here γ_0 and γ_1 are the usual first two terms in the Gell-Mann-Low renormalization group function. The parameter Λ_0 is an integration constant of the renormalization group equation. It will set the overall scale of the strong interactions, and will cancel from dimensionless ratios. We now take this well known dependence and solve it for the cutoff as a function of the coupling. Inserting the result in Eq. (1) shows how the correlation length diverges as we approach the critical point at vanishing coupling

$$\xi^{-1} = \frac{m}{\Lambda_0} (\gamma_0 g_0^{\,2})^{-\gamma_1/2\gamma_0^2} \exp\left(\frac{-1}{2\gamma_0 g_0^{\,2}}\right) \times \left(1 + O(g_0^{\,2})\right) \quad . \tag{3}$$

Thus, to determine a mass in units of the integration constant Λ_0, first determine by some method such as Monte Carlo simulation the divergence of the correlation length as the coupling is reduced. The coefficient of this divergence is the desired ratio.

The important observation at this point is that Λ_0 is independent of the particular mass being measured. One could first use the

correlation between operators with, say, ρ quantum numbers, and then the correlation between two proton operators. Taking the ratio of the coefficients of the corresponding divergences gives the ρ to proton mass ratio with no parameters left to adjust beyond the bare quark masses, which are already determined most naturally from the pseudoscalar masses.

To illustrate the procedure, consider the long range linear potential between quark-like sources in the pure SU(3) gauge theory. In Fig. 1 I show Monte Carlo measurements of the effective force, in lattice units, between two such sources at various separations [3]. The points form an envelope representing the strength of the constant long range force K in units of the lattice spacing a. This force is plotted versus $\beta = 6/g_0^{\,2}$ so that the dominant exponential behavior in Eq. (3) appears as a nearly straight line when plotted on this logarithmic paper. The normalization gives the string tension in units of the square of the Λ parameter, the band plotted in the figure representing

$$\Lambda_0 = (6 \pm 1) \times 10^{-3}\sqrt{K} \quad . \tag{4}$$

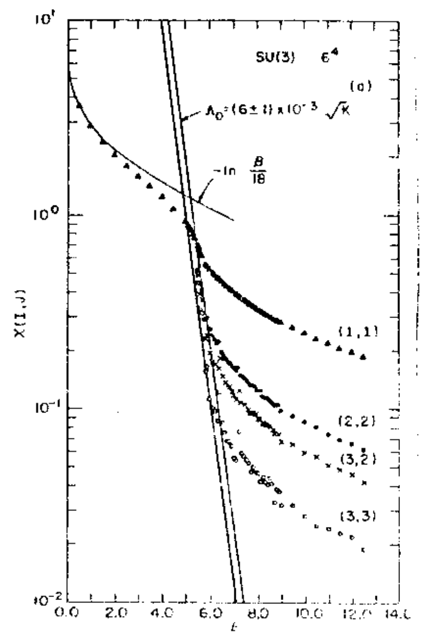

Fig. 1

The effective force between quarks in pure SU(3) lattice gauge theory.
The χ ratios are defined in Ref. [3]. Their envelope is the coefficient of the long range force, measured in lattice units.

Correcting for scheme dependence, and putting in the phenomenological value $\sqrt{K} \approx 400$ (MeV), this corresponds to the more conventional Λ_{MOM} being about 180 MeV.

In this way one can attempt to calculate from first principles the physical observables of the strong interactions. Note that the essential singularity of Eq. (3) is inherently nonperturbative. As mentioned above, this is one of the prime motivations for the lattice cutoff. In lattice gauge theory the variables are elements of the gauge group and are usually associated with the bonds of a hypercubical lattice. These represent the familiar path ordered exponentials between the corresponding sites.

Several techniques have been developed for calculations in lattice gauge theory. As the lattice is just a cutoff, one could proceed with ordinary perturbation theory. This should reproduce all the standard results as obtained with other regulators. The lattice propagators are, however, rather complicated, and thus only a few one loop calculations have been done [4]. As other schemes for perturbative analysis are rather highly developed, the value of the lattice lies elsewhere.

In his initial work on the subject, Wilson studied the strong coupling limit of the theory [5]. High temperature expansions from statistical mechanics are directly applicable here, and confinement is readily derived. Indeed, the theory reduces to one of quarks on the ends of thin strings with a finite energy per unit length. Unfortunately, as discussed above, we must leave strong coupling for the continuum limit. Mean field theory analysis, which becomes exact as the dimension of space-time becomes large, suggests the possibility of a deconfining phase transition as the coupling is reduced [6]. Duality arguments show that such a transition does exist in simple toy models based on discrete gauge groups [6]. Thus we must worry whether confinement persists in the continuum limit.

Before the Monte Carlo evidence, the strongest arguments for confinement came from the Migdal-Kadanoff approximate recursion relations [7]. These showed a strong analogy between spin systems in two dimensions and gauge theories in four. As two dimensional spin systems with a non-Abelian symmetry are believed not to have a spin wave phase, this suggests that the corresponding four dimensional gauge theories are confining for all values of coupling. Although we now know that these approximations to the renormalization group functions have some

shortcomings, they have had considerable success in predicting gross phase structure and should receive more attention [8]. A recent paper of Tomboulis [9] argues that these relations provide a lower bound to the interquark potential.

This brings me to Monte Carlo simulation, which has become by far the most popular calculational tool in this field [10]. To the particle physicist, Monte Carlo provides a technique for the numerical evaluation of path integrals. The method converges reasonably well in all domains of coupling and in principle permits the evaluation of any desired correlation. The calculations in practice, however, do have inherent limitations. First of all, the lattices being four dimensional are necessarily rather limited in size, typically of order 10 sites on a side. Thus both finite volume and finite lattice spacing effects must be carefully monitored and traded off against each other. Nevertheless, the experimental observation of precocious scaling in deeply inelastic leptonic scattering suggests that a few qualitative results are possible. Secondly, statistical errors decrease only with the square root of the computer time. In particular, this has plagued the analysis of glueball masses.

Monte Carlo calculations have given two rather clear and uncontroversial numbers. The first of these is the ratio of the string tension to the asymptotic freedom scale mentioned above. The second number is the temperature of the physical transition where confinement loses meaning because the vacuum becomes a soup of gluonic flux [11]. Let me note in passing that this transition appears to be quite abrupt, probably first order, with a nearly empty vacuum going suddenly to an essentially free gas of quarks and gluons. This abruptness may be related to the successes of the bag model [12], which would suggest a transition at the temperature where the pressure of a free gluon gas equals the bag constant. The estimated deconfinement transition temperature of 180 MeV would then give a bag constant of

$$B = \frac{8}{3} \sigma T_c^4 = (8\pi^2 T_c^4/45) \approx (210 \text{ MeV})^4 \quad . \tag{5}$$

Another observable that has received considerable attention is the glueball mass. Indeed, a whole spectrum of states is being investigated, primarily by the two groups in Ref. [13]. Schierholz's talk at this conference reviews the status of these calculations. Let me only remark that the lightest 0^{++} state is reasonably uncontroversially found at 700–1000 MeV, and various methods indicate an extremely rich spectrum lies below 2 GeV.

How to include quarks in the lattice calculation remains an active and unsettled question. From an analytic point of view, a fermionic path integral is perfectly well defined; however, it is not an integral in the classical sense and thus it is unclear how to use importance sampling techniques for numerical work. Analytically removing the fermionic fields leaves an ordinary integral over the gauge fields but the weight includes the determinant of an enormous matrix. The Monte Carlo method needs numerical estimates for the changes in this determinant as the gauge fields are altered. This requires knowledge of its inverse, and several schemes for this evaluation have been proposed. In an early paper, Weingarten and Petcher [14] proposed directly evaluating the inverse with a Gauss-Seidel procedure. This involves rather intensive computation, although they were able to carry out calculations on a 2^4 site lattice. Fucito, Marinari, Parisi and Rebbi [15] proposed using Monte Carlo with auxiliary scalar fields to find the required inverse stochastically. Recent calculations [16] with this method are encouraging, indicating that the effects of the fermionic loops are controllable. Kuti [17] has recently proposed the use of a stochastic method of Von Neuman and Ulam for the inverse. Results from this technique with gauge theories are eagerly anticipated. As an estimate of the effects of the virtual quark loops, Ref. [18] has compared calculations with −2 and 0 flavors, the former obtained from a conventional Monte Carlo simulation with commuting fields. Their results indicate that hadronic masses are fairly stable, but that decay constants can receive substantial corrections.

The most publicized results with quark fields involve the "valence" or "quenched" approximation, wherein the quark propagators are calculated in a gauge field configuration obtained without any feedback from the quarks [19]. This amounts to neglecting diagrams containing internal fermionic loops. The results support the existence of chiral symmetry breaking and give a qualitative hadronic spectrum. There have, however, been some indications that systematic effects have been underestimated. See in particular Schierholz's talk at this conference.

At this point let me change the subject somewhat and discuss a few alternative simulation techniques which may have certain advantages

over the usual procedures. In Ref. [20] U(1) lattice gauge theory was studied with an analog of a molecular dynamics calculation. In addition to the gauge fields U_{ij} on the lattice links, the authors also introduced conjugate momenta ℓ_{ij} and studied the classical evolution in a new time dimension of the four dimensional lattice with Hamiltonian

$$H = S(U) + 1/2 \sum_{\{i,j\}} \ell_{ij}^2 \quad . \qquad (6)$$

This sets up a meandering through phase space on the surface of constant total "energy". If the behavior is ergodic, this will sample the microcanonical ensemble. Note that the procedure contains neither random numbers nor a temperature parameter. The randomness arises from the complexity of the system itself, and the temperature can be measured from the average kinetic energy

$$1/2\ kT = 1/2 \langle \ell_{ij}^2 \rangle \quad . \qquad (7)$$

In simple cases one can dispense with the extra variables ℓ_{ij} and set up a random walk through configurations of a given total action. This would represent a hybrid of the microcanonical and Monte Carlo methods. Consider for example SU(2) lattice gauge theory. When looking at a given link, the action would be unchanged if that variable were replaced with another SU(2) matrix on a spherical submanifold of the full SU(2) group. This suggests a simulation algorithm wherein one sweeps through lattice and replaces each group element with another from the corresponding sphere. This has the advantage over the differential equation approach that it allows large changes in the elements. Empirically, the algorithm seems to converge comparably to a well optimized conventional Monte Carlo approach. It has the disadvantage that extracting the temperature is not straightforward. This is not a problem for the particle theorist because the bare coupling is not a physical observable. To test scaling laws he can use any quantity which perturbatively begins as the coupling squared, for example the internal energy, which is exactly constant in this algorithm.

This procedure takes special advantage of the shape of the SU(2) manifold; indeed, generalization to other groups is unclear. I will now present a new microcanonical simulation technique which is easily applied to any type of variable [21]. It again consists of a random walk through configuration space while maintaining a constraint on the total energy. A single extra degree of freedom, a "demon", travels around the system, transferring energy as he changes the dynamical variables. This demon is analogous to the kinetic energy in the molecular dynamics method, except that he is not associated with any particular lattice variable. This demon carries a sack of energy with non negative contents. Upon visiting a variable, he attempts to change it as in a usual simulation. Instead of accepting or rejecting the change based on random numbers, he makes the change if he has sufficient energy in his sack. The latter is adjusted so that the demon's energy plus that of the lattice system remains unchanged. As the demon is essentially a free variable, his average energy is a measure of the temperature

$$\langle E_D \rangle = 1/\beta \qquad (8)$$

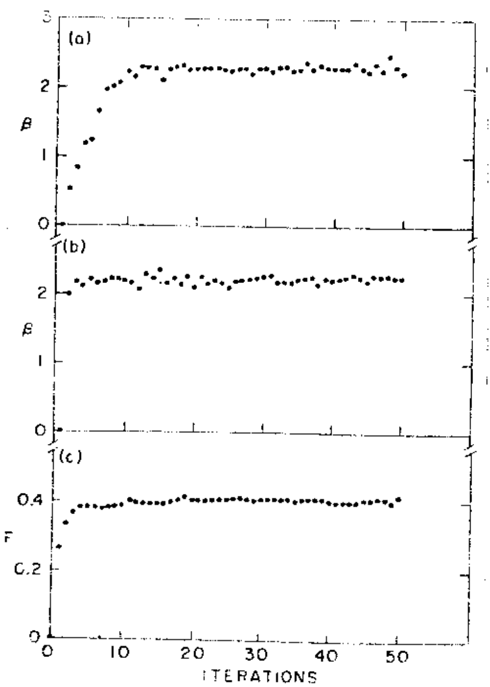

Fig. 2

Three simulations of SU(2) lattice gauge theory on a 6^4 lattice.
In (a) a demon with a large initial energy store moves sequentially through an initially ordered lattice.
In (b) he hops randomly through the system.
For comparison, (c) shows a conventional simulation at $\beta = 2.25$.

The demon might be regarded has a heat bath, but one with a very small heat capacity. If instead of a single demon, one releases a whole battalion, then they can contain an appreciable amount of energy. When the number of demons becomes large compared to the number of lattice degrees of freedom, the simulation reduces to the conventional Metropolis et al. procedure. Fig. 2 shows the results of two SU(2) simulations with this procedure and one conventional simulation.

This algorithm has some advantages. First, the demon has no need for transcendental functions; his energy becomes automatically exponentially distributed. Secondly, he is rather lenient in his demands for quality random numbers. In the case of a simulation of Ising variables, no random numbers are needed at all; the lattice uses its complexity to generate its own. Third, for discrete groups all arithmetic can be done with small integers. This means that several demons could ride on one computer word and all processing of several spins can occur simultaneously. Finally, the method does not treat the Boltzmann weight as a probability; indeed, temperature does not even appear as a parameter. As one of the problems with simulating fermionic systems is the lack of a probability interpretation for the path integral, perhaps there is a hint here.

To conclude this talk, let me observe that Monte Carlo techniques for gauge theories seem to be rapidly reaching technological limits. The glueball and fermionic calculations are using hundreds of hours on state-of-the-art computers. In contrast, note that the analytic techniques for lattice gauge theory have been relatively neglected. Whereas a few tens of particle physicists have actively pursued strong coupling expansions or Migdal-Kadanoff recursion relations, an order of magnitude more have worked on the numerical simulations. This suggests that more balance and perhaps hybrid techniques are in order. In addition, I feel that we are a long way from the last word on techniques for dealing with fermions.

ACKNOWLEDGMENT

This work has been authored under DOE Contract #DE-AC02-76CH00016.

REFERENCES

[1] S. Coleman and E. Weinberg, Phys. Rev. D7 (1973) 1888.

[2] D. Gross and F. Wilczek, Phys. Rev. Lett. 30 (1973) 1343; Phys. Rev. D8 (1973) 3633;
W.E. Caswell, Phys. Rev. Lett. 33 (1974) 244;
D.R.T. Jones, Nucl. Phys. B75 (1974) 531.

[3] M. Creutz and K.J.M. Moriarty, Phys. Rev. D26 (1982) 2166.

[4] A. Hasenfratz and P. Hasenfratz, Phys. Lett. 93B (1980) 165.

[5] K. Wilson, Phys. Rev. D10 (1974) 2445.

[6] R. Balian, J.M. Drouffe, C. Itzykson, Phys. Rev. D10 (1974) 3376; D11 (1975) 2098; D11 (1975) 2104.

[7] L.P. Kadanoff, Rev. Mod. Phys. 49 (1977) 267.

[8] K.M. Bitar, S. Gottlieb, C. Zachos, Phys. Rev. D26 (1982) 2853.

[9] E. Tomboulis, Phys. Rev Lett. 50 (1983) 885.

[10] M. Creutz, L. Jacobs, C. Rebbi, Physics Reports (in press).

[11] L. McClerran and B. Svetitsky, Phys. Lett. 98B (1981) 195;
J. Kuti, J. Polonyi, K. Szlachanyi, Phys. Lett. 98B (1981) 199;
K. Kajantie, C. Montonen, E. Pietarinen, Zeit. Phys. C9 (1981) 253;
J. Engels, F. Karsch, H. Satz, J. Montvay, Phys. Lett. 101B (1981) 89; Nucl. Phys. B205 (1982) 545.

[12] K. Johnson, talk at this conference.

[13] B. Berg and A. Billoire, Phys. Lett. 114B (1982) 324;
K. Ishikawa, G. Schierholz, M. Teper, Phys. Lett. 110B (1982) 399.

[14] D. Weingarten and D. Petcher, Phys. Lett. 99B (1981) 333.

[15] F. Fucito, E. Marinari, G. Parisi, C. Rebbi, Nucl. Phys. B180 (1981) 369.

[16] H. Hamber, E. Marinari, G. Parisi, C. Rebbi, preprint (1983).

[17] J. Kuti, Phys. Rev. Lett. 49 (1982) 183.

[18] W. Duffy, G. Guralnik, D. Weingarten, preprint (1983)

[19] D. Weingarten, Phys. Lett. 109B (1982) 57;
H. Hamber and G. Parisi, Phys. Rev. Lett. 47 (1982) 1792;
E. Marinari, G. Parisi, C. Rebbi, Phys. Rev. Lett. 47 (1981) 1798.

[20] D. Callaway and A. Rahman, Phys. Rev. Lett. 49 (1982) 613.

[21] M. Creutz, preprint (1983).