Monte Carlo studies of Wilson loops in SU(5) gauge theory in four dimensions

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Monte Carlo simulations on a 6^4 lattice are used to calculate Wilson loops, and hence the string tension, for pure SU(5) gauge theory. A bound is obtained for the string tension.

In some recent papers, we studied pure SU(3) (Refs. 1 and 2) and SU(4) (Refs. 3 and 4) gauge theories with the Wilson action on a hypercubical lattice in four space-time dimensions. The Wilson loops were measured, the string tension was presented, and the corresponding asymptotic-freedom scale parameters Λ_0 were measured. In the present paper, we wish to extend these measurements to the gauge group SU(5). As a result of this analysis, we would like to establish that SU(5) color is confined and show how Λ_0, the asymptotic-freedom scale parameter, varies with the number of colors N. The first-order transition in the theory tends to mask the asymptotic-freedom scaling and thus we are able only to put a bound on the string tension.

The details of our calculational procedure have already been presented in the literature[5,6] and so will not be repeated here. A hypercubical lattice in four Euclidean dimensions was used in our study. Between the nearest-neighbor lattice sites denoted by i and j we form a link $\{i,j\}$ to which is attached an $N\times N$ unitary-unimodular matrix U_{ij} of the group SU(N) such that

$$U_{ji}=(U_{ij})^{-1} .$$

Our partition function is defined by

$$Z(\beta)=\int\left[\prod_{\{i,j\}} dU_{ij}\right]\exp(-\beta S[U]) ,$$

where β is directly proportional to the inverse coupling constant squared, $\beta=2N/g_0^2$, with g_0 the bare coupling constant. The measure dU_{ij} in the definition of the partition function is the normalized invariant Haar measure for SU(N). We define our action S as the sum over all unoriented plaquettes $\Box$ such that

$$S[U]=\sum_{\Box} S_{\Box}=\sum_{\Box}\left[1-\frac{1}{N}\operatorname{Re}\operatorname{Tr}U_{\Box}\right] ,$$

where $U_{\Box}$ is the parallel transporter around a plaquette. We imposed periodic boundary conditions throughout our calculations and equilibriated our lattice by the method of Metropolis *et al.*[7] In order to make our calculations manageable, they are carried out on the CDC CYBER 205, a pipelined vector processor. For the rest of this paper, we will discuss only SU(5).

Rectangular Wilson loops[8] are defined by the expectation value

$$W(I,J)=\tfrac{1}{5}\langle \operatorname{Re}\operatorname{Tr}U_C\rangle ,$$

where C is a rectangle of length I and width J and U_C is the parallel transporter around C. The leading-order strong- and weak-coupling expansions for the Wilson loop are

$$W(I,J)=\left[\frac{\beta}{50}\right]^{IJ}[1+O(\beta^2)] \tag{1}$$

and

$$W(1,1)=\frac{6}{\beta}+O(\beta^{-2}) , \tag{2}$$

respectively. In order to extract the string tension we form the logarithmic ratios $\chi(I,J)$ defined by

$$\chi(I,J)=-\ln\left[\frac{W(I,J)\,W(I-1,J-1)}{W(I,J-1)\,W(I-1,J)}\right] .$$

The relationship between the string tension K, the coefficient of the area term in the Wilson loops, and the logarithmic ratios $\chi(I,J)$ is

$$\chi(I,J)=\frac{K}{\Lambda_0^2}\left[\frac{275}{24\pi^2\beta}\right]^{-102/121}\exp\left[-\frac{24\pi^2\beta}{275}\right] , \tag{3}$$

where a parameter Λ_0, called the asymptotic-freedom scale parameter, has been introduced. The leading-order strong-coupling expansion for the string tension is given by

$$\chi(I,J)=-\ln\left[\frac{\beta}{50}\right]+O(\beta^2) . \tag{4}$$

In Fig. 1 we show the Wilson loops up to size 3×3. Our calculations were performed by first carrying out 300 iterations through the 6^4 lattice with 25 Monte Carlo upgrades per link. This equilibriated our space-time lattice. The Wilson loops were then obtained by averaging over the next 100 iterations through the lattice. However, every second

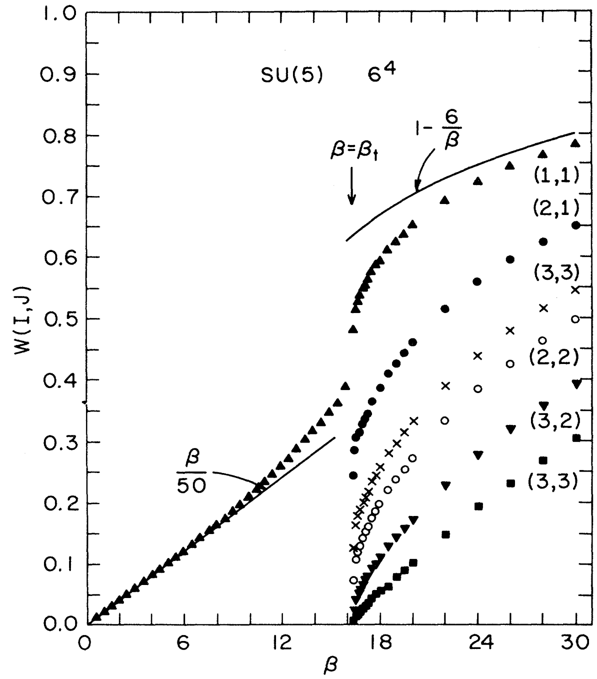

FIG. 1. The Wilson loops $W(I,J)$ for pure SU(5) gauge theory on a 6^4 lattice as a function of the inverse coupling constant squared β. The solid upward triangles represent $(I,J)=(1,1)$, the solid circles represent (2,1), the crosses represent (2,2), the solid downward triangles represent (3,2), and the squares represent (3,3). The curves represent the leading-order strong- and weak-coupling expansions of Eqs. (1) and (2), respectively.

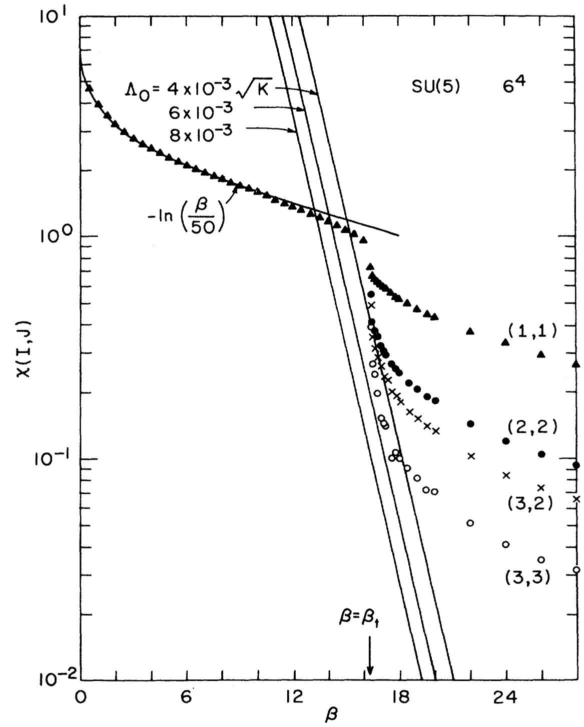

FIG. 2. The string tension $\chi(I,J)$ for pure SU(5) gauge theory on a 6^4 lattice as a function of the inverse coupling constant squared β. The solid triangles represent $(I,J)=(1,1)$, the solid circles represent (2,2), the crosses represent (3,2), and the open circles represent (3,3). Also shown in the diagram is the leading-order strong-coupling expansion of Eq. (4).

iteration through the lattice was ignored in order to reduce the correlation between iterations. Thus, 50 lattice configurations were used in our averages. One upgrade per link took 833 μsec on the CDC CYBER 205. The transition point for pure SU(5) gauge theory has previously been measured[9,10] to be $\beta_t=16.3\pm0.3$. As a result, we used ordered starting lattices for $\beta>\beta_t$ and disordered starting lattices for $\beta<\beta_t$. In Fig. 1, we also show the leading-order strong- and weak-coupling expansions of Eqs. (1) and (2), respectively.

We present the logarithmic ratios $\chi(I,J)$ for $(I,J)=(1,1)$, (2,2), (3,2), and (3,3) as a function of the inverse coupling constant squared β in Fig. 2. The leading-order strong-coupling expansion of Eq. (4) agrees with the data up to $\beta\simeq10.0$. In this figure, we show curves which correspond to the behavior of Eq. (3) with $\Lambda_0=4,6$ and $8\times10^{-3}\sqrt{K}$. Note that even for the largest loops the desired scaling is not observed. The first-order transition results in a rapid rise of the χ ratios as a lattice artifact. We interpret this behavior as giving an upper bound on the continuum string tension of

$$\Lambda_0\geqslant5\times10^{-3}\sqrt{K}\quad .$$

In Fig. 3 we present the asymptotic-freedom scale parameters Λ_0, for SU(N) as a function of $1/N$ for $N=2$, 3, 4, and 5. In a recent quenched reduced Eguchi-Kawai calculation[11] it was estimated that the Λ_0 parameter for SU(∞) is

$$\Lambda_0=(2\pm1)\times10^{-3}\sqrt{K}\quad ,$$

which is also shown in our diagram. Our results for finite N suggest that this may be artificially small due to artifacts of the first-order transition.

We note that in an analysis of the SU(2) model with both fundamental and adjoint couplings, Bhanot and Dashen[12] found marked deviations from expected perturbative

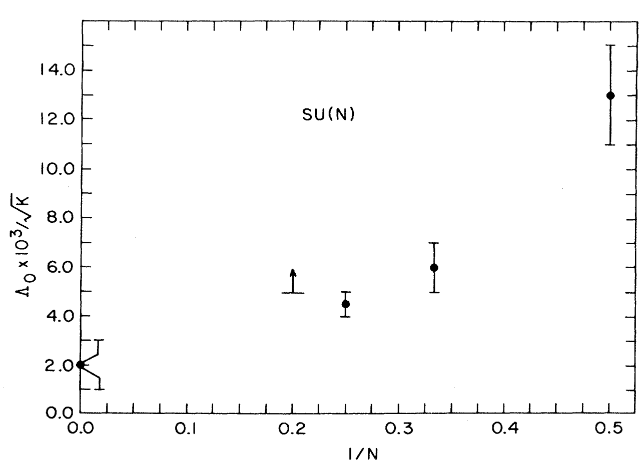

FIG. 3. The asymptotic-freedom scale parameters Λ_0 for SU(N) gauge theory as a function of $1/N$.

behavior when working in a coupling region near the first-order transition line appearing with positive adjoint coupling. The large-N SU(N) transitions have been interpreted in terms of this line extending across the Wilson axis with the larger groups. Thus, the fact that we do not see the desired asymptotic scaling with the SU(5) model is probably closely related to the improper scaling seen in Ref. 12 near the extraneous critical point.

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[1]M. Creutz and K. J. M. Moriarty, Phys. Rev. D 26, 2166 (1983).
[2]R. W. B. Ardill, M. Creutz, and K. J. M. Moriarty, Phys. Rev. D 27, 1956 (1983).
[3]D. Barkai, M. Creutz, and K. J. M. Moriarty, Nucl. Phys. B (to be published).
[4]D. Barkai, M. Creutz, and K. J. M. Moriarty, Phys. Lett. B (to be published).
[5]D. Barkai and K. J. M. Moriarty, Comput. Phys. Commun. 25, 57 (1982); 26, 477 (1982); 27, 105 (1982).
[6]D. Barkai, M. Creutz, and K. J. M. Moriarty, Comput. Phys. Commun. 30, (1983).
[7]N. Metropolis, A. W. Rosenbluth, M. N. Rosenbluth, A. H. Teller, and E. Teller, J. Chem. Phys. 21, 1087 (1953).
[8]K. G. Wilson, Phys. Rev. D 10, 2445 (1974).
[9]M. Creutz, Phys. Rev. Lett. 46, 1441 (1981).
[10]H. Bohr and K. J. M. Moriarty, Phys. Lett. 104B, 217 (1981).
[11]G. Bhanot and K. J. M. Moriarty, Phys. Lett. 122B, 271 (1983).
[12]G. Bhanot and R. Dashen, Phys. Lett. 113B, 299 (1982).