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LETTER TO THE EDITOR

Numerical studies of Wilson loops in SU(6) gauge theory in four dimensions

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Abstract. A previous Monte-Carlo study of pure SU(6) gauge theory in four space–time dimensions is extended to measure all Wilson loops up to size 3×3 on a 6^4 lattice. The string tension is extracted from the Wilson loops and a lower bound is placed on the asymptotic-freedom scale parameter Λ_0.

In a recent paper (Creutz and Moriarty 1982), pure SU(6) gauge theory with the Wilson action was studied on a hypercubical lattice in four space–time dimensions. This study established that SU(6) gauge theory has a first-order phase transition. The strong-coupling expansion (Drouffe and Moriarty 1982) was shown to fit the Monte-Carlo data over the whole strong-coupling region. We would now like to study the approach of pure SU(6) gauge theory to asymptotic-freedom scaling. With this in view, we calculated all Wilson loops up to size 3×3 on a 6^4 lattice and determined the coefficient of the area term in these loops. Unfortunately, because of the first-order phase transition, asymptotic-freedom scaling can only set in for Wilson loops larger than 3×3. Hence we are only able to put a lower bound on the asymptotic-freedom scaling parameter Λ_0.

We use a hypercubical lattice in four Euclidean space–time dimensions in our calculations. Denoting the nearest-neighbour lattice sites by i and j, a link $\{i, j\}$ is formed on which sits an $N\times N$ unitary-unimodular matrix $U_{ij}\in \mathrm{SU}(N)$ with

$$U_{ji}=(U_{ij})^{-1}.$$

Our partition function is written as

$$Z(\beta)=\int\left(\prod_{\{i,j\}} \mathrm{d}U_{ij}\right)\exp(-\beta S[U])$$

where $\beta=2N/g_0^2$, with g_0 the bare coupling constant and the measure in the above integral is the SU(N) normalised invariant Haar measure. Our action S is defined as the sum over

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all unoriented plaquettes □ with

$$S[U]=\sum_{\Box} S_{\Box}=\sum_{\Box}\left(1-\frac{1}{N}\,\mathrm{Re}\,\mathrm{Tr}\,U_{\Box}\right)$$

where $U_{\Box}$ is the product of link variables around a plaquette. Periodic boundary conditions were used throughout our calculations and our lattice was equilibriated by the method of Metropolis *et al* (1953). Because of the amount of statistics required and the large lattice on which we are working, we resorted to a pipelined vector processor, the CDC CYBER 205, for our calculations. Our calculational methods on this computer have recently been described in great detail (Barkai and Moriarty 1982, Barkai *et al* 1983a). From now on we specialise to SU(6).

We define our rectangular Wilson loops (Wilson 1974) by

$$W(I, J)=\tfrac{1}{6}\langle \mathrm{Re}\,\mathrm{Tr}\,U_{\mathrm{C}}\rangle$$

where C denotes a rectangle of length I and width J and U_{C} is the product of link variables around C. The Wilson loops have the leading-order strong-coupling (Drouffe and Moriarty 1982) and weak-coupling expansions

$$W(I, J)=(\tfrac{1}{72}\beta)^{IJ}(1+\mathrm{O}(\beta^2)) \tag{1}$$

and

$$W(I, J)=\frac{35}{4\beta}+\mathrm{O}(\beta^{-2}) \tag{2}$$

respectively. We extract the string tension by forming the logarithmic ratios $\chi(I, J)$ defined by

$$\chi(I, J)=-\ln\left(\frac{W(I, J)W(I-1, J-1)}{W(I, J-1)W(I-1, J)}\right).$$

The coefficient of the area term in the Wilson loops, K, is related to the logarithmic ratios $\chi(I, J)$ by

$$\chi(I, J)=\frac{K}{\Lambda_0^2}\left(\frac{33}{2\pi^2\beta}\right)^{(-102/121)}\exp\left(-\frac{2\pi^2\beta}{33}\right) \tag{3}$$

where Λ_0 is the asymptotic-freedom scale parameter. The leading-order strong-coupling expansion for the string tension is given by

$$\chi(1, 1)=-\ln(\tfrac{1}{72}\beta)+\mathrm{O}(\beta^2). \tag{4}$$

All the Wilson loops up to size 3 × 3 are shown in figure 1. To perform our calculations we first carried out 300 iterations through the 6^4 lattice with 36 Monte-Carlo updates per link. These iterations were thrown away and were used merely to equilibriate our lattice. The Wilson loop averages were then obtained from the next 100 iterations through the lattice. In order to cut down correlations between sweeps, we ignored every second lattice configuration and so only 50 lattice configurations were used in our averages. One update per link took 1.91 ms on the CDC CYBER 205. By observing the standard deviation on the Wilson loops, we measured the transition point for pure SU(6) gauge theory to be $\beta_{\mathrm{t}}=23.8\pm0.02$. Therefore, ordered starting lattices were used for $\beta>\beta_{\mathrm{t}}$ and disordered

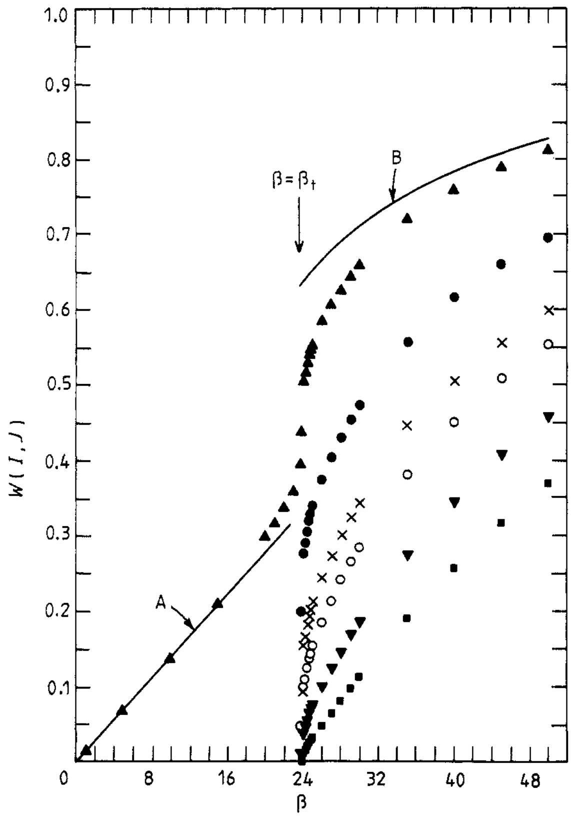

Figure 1. The Wilson loops $W(I, J)$ for pure SU(6) gauge theory on a 6^4 lattice as a function of the inverse coupling constant squared β. ▲, $(I, J)=(1, 1)$; ●, $(I, J)=(2, 1)$; ×, $(I, J)=(3, 1)$; ○, $(I, J)=(2, 2)$; ▼, $(I, J)=(3, 2)$; ■, $(I, J)=(3, 3)$. The curves A and B represent the leading-order strong- and weak-coupling expansions of equations (1) and (2), respectively.

starting lattices were used for $\beta<\beta_t$. The leading-order strong- and weak-coupling expansions of equations (1) and (2), respectively, are also presented in figure 1.

In figure 2 we present the logarithmic ratios $\chi(I, J)$ for $(I, J)=(1, 1)$, (2, 2), (3, 2) and (3, 3) as a function of the inverse coupling constant squared β. Also shown in figure 2 are curves corresponding to the behaviour of equation (3) with $\Lambda_0=4$, 6 and $8\times10^{-3}\sqrt{K}$. We can see that even for loops up to size 3×3, asymptotic-freedom scaling is not observed because of the first-order phase transition. This only allows us to place a lower bound on the asymptotic-freedom scale parameter of

$$\Lambda_0 \geqslant (5\times10^{-3})\sqrt{K}.$$

A similar result was recently found for SU(5) gauge theory (Barkai *et al* 1983b). Also shown in figure 2 is the leading-order strong-coupling expansion of equation (4).

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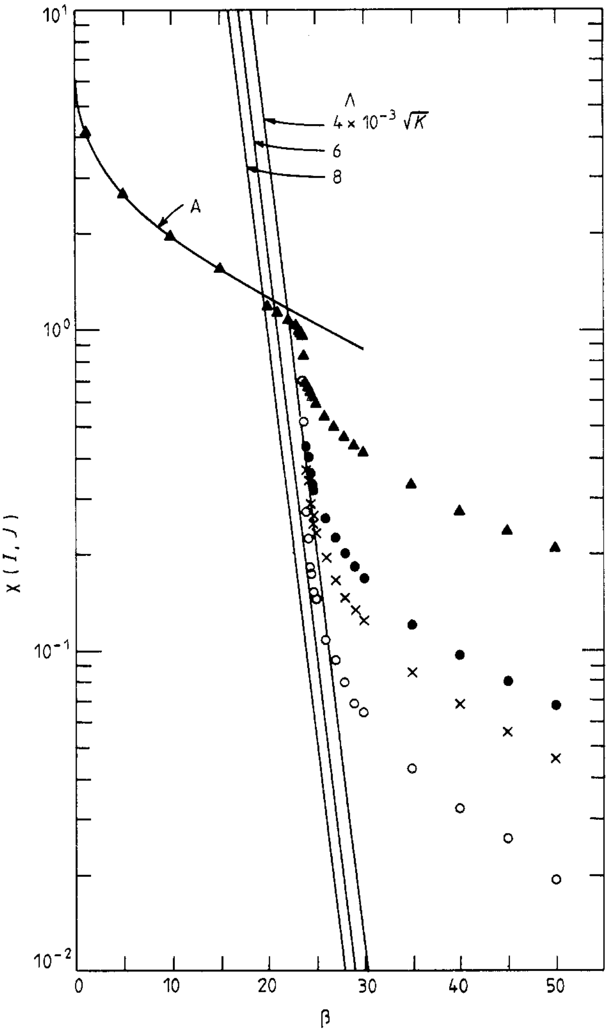

Figure 2. The string tension $\chi(I, J)$ for pure SU(6) gauge theory on a 6^4 lattice as a function of the inverse coupling constant squared β. ▲, $(I, J)=(1, 1)$; ●, $(I, J)=(2, 2)$; ×, $(I, J)=(3, 2)$; ○, $(I, J)=(3, 3)$. Also shown in the diagram (curve A) is the leading-order strong-coupling expansion of equation (4).

References

Barkai D, Creutz M and Moriarty K J M 1983a *Comput. Phys. Commun.* **30** 13

—— 1983b Monte Carlo Studies of Wilson Loops in SU(5) Gauge Theory in Four Dimensions *Brookhaven National Laboratory Preprint*

Barkai D and Moriarty K J M 1982 *Comput. Phys. Commun.* **27** 105

Creutz M and Moriarty K J M 1982 *Phys. Rev.* D **25** 1724

Drouffe J-M and Moriarty K J M 1982 *Phys. Lett.* **108B** 333

Metropolis N, Rosenbluth A W, Rosenbluth M N, Teller A H and Teller E 1953 *J. Chem. Phys.* **21** 1087

Wilson K G 1974 *Phys. Rev.* D **10** 2445