

Modified Wilson action and Z_2 artifacts in SU(2) lattice gauge theory

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A modified Wilson action with an additional chemical potential for the number of “negative plaquettes” is used to study the role of Z_2 artifacts in SU(2) lattice gauge theory and their possible influence on large-scale physics. The phase diagram of the model with modified action is studied. We show that large-scale objects, e.g., Wilson loops and their ratios, can be strongly influenced by lattice artifacts.

The lattice approach has opened the possibility to use powerful numerical methods for nonperturbative study of gauge theories. One of the most popular choices of pure gauge lattice action is the one proposed by Wilson [1] [for the SU(2) group]:

$$S_W(U_l) = \beta \sum_{\square} (1 - \frac{1}{2} \text{Tr} U_{\square}), \tag{1}$$

where $\beta = 4/g_{\text{bare}}^2 \equiv 4/g^2$, $U_l \equiv U_{\text{link}} \in \text{SU}(2)$ are the field variables defined on the links $l \equiv (x, \mu)$, $\square \equiv (x; \mu\nu)$ refers to the location and orientation of the corresponding plaquette, and U_{\square} are plaquette variables:

$$U_{\square} \equiv U_{x;\mu\nu} = U_{x;\mu} U_{x+\mu;\nu} U_{x+\nu;\mu}^{\dagger} U_{x;\nu}^{\dagger}. \tag{2}$$

The average of any field functional $\mathcal{O}(U)$ is defined as

$$\langle \mathcal{O}(U) \rangle = Z^{-1} \int \prod_{\text{links}} dU_l \mathcal{O}(U) \exp[-S_W(U_l)], \tag{3}$$

where

$$Z = \int \prod_{\text{links}} dU_l \exp[-S_W(U_l)], \tag{4}$$

with some choice of boundary conditions (periodic in our case).

In the Monte Carlo approach, to calculate the average of any field operator one must generate a sequence of (equilibrium) field configurations: $(U_{\text{link}}^{(1)}), (U_{\text{link}}^{(2)}), \dots$, with the weight $P(U_l) \sim \exp[-S_W(U_l)]$ and average over all of them.

The crucial question in this approach is whether these field configurations provide an adequate representation of continuum physics (at least, at moderately large values of β). The well-known fact is that lattice theories can suffer from lattice artifacts. One example of the influence of such lattice artifacts is provided by the study of the topological properties of gauge theories.

Given the geometric approach to the definition of the topological charge Q_i [2,3], the topological susceptibility

χ_i is defined as $\chi_i = \langle Q_i^2 \rangle / V$, V being the space-time volume.

For any given lattice configuration (U_{link}) the topological charge Q_i is the sum over all lattice sites: $Q_i = \sum q[U]$, where for smooth enough fields

$$q[U] = -\frac{a^4}{32\pi^2} \epsilon_{\mu\nu\rho\sigma} \text{Tr}(F_{\mu\nu} F_{\rho\sigma}) + O(a^6), \tag{5}$$

and, therefore, the value $q[U]$ has the correct classical continuum limit. The crucial element is the notion of a *smooth* field. In the quantized theory the contribution of nonsmooth (rough) field configurations must be taken into account. For example, small-scale fluctuations, i.e., fluctuations living on the scale size of, say, one lattice spacing, and carrying a nontrivial topological charge (so-called dislocations) can lead to the divergence of the topological susceptibility in the continuum limit (at least for this geometric definition of topological charge). One source of trouble is the special field configurations containing negative plaquettes: $\frac{1}{2} \text{Tr} U_{\square} \simeq -1$, [4–6]. A similar situation takes place for the CP^{n-1} model with $n \leq 3$ [7].

Rough fluctuations can exhibit themselves in the form of Z_2 strings and Z_2 monopoles [8–13]. Z_2 strings consist of sequences of plaquettes where

$$\text{sgn}(\text{Tr} U_{\square}) = -1 \tag{6}$$

and a Z_2 monopole is attached to a three-dimensional cube c if

$$\prod_{\square \in c} \text{sgn}(\text{Tr} U_{\square}) = -1. \tag{7}$$

As was shown in Ref. [10], the number of Z_2 monopoles decreases exponentially with increasing β :

$$N_{Z_2}(\beta) \sim e^{-c_{Z_2} \beta}, \tag{8}$$

where $c_{Z_2} \simeq 2$. But this in itself does not mean that they become unimportant at large β . They can still strongly influence any lattice average $\langle \mathcal{O}(U) \rangle(\beta)$ (say, Wilson loops) which decays faster than $N_{Z_2}(\beta)$ with increasing β , and, so, even a small admixture of these rapidly varying fields can become competitive. Note also that the density of the Z_2 monopoles $\rho_{Z_2} \sim N_{Z_2}(\beta)/a(\beta)^3$ tends to infinity in the continuum limit if one uses the standard two-loop renormalization-group expression for the lattice spacing $a(\beta)$.

It is very important to study the role of the underlying Z_2 degrees of freedom to separate the "real" (large-scale) physics from lattice artifacts. To do it one can modify the lattice action in such a way that small-scale fluctuations (with negative plaquettes) would be suppressed without positive plaquettes being touched. So, the simplest choice of modified action S serving this goal is

$$S = S_W + S_\lambda, \quad (9)$$

where S_W is the standard Wilson action and S_λ is the additional term which suppresses the negative plaquettes:

$$S_\lambda = \lambda \sum_{\square} [1 - \text{sgn}(\text{Tr} U_{\square})]. \quad (10)$$

The parameter λ plays the role of a chemical potential for the number of negative plaquettes. [One can choose $\lambda \equiv \lambda(\beta)$ in the form $\lambda \equiv \lambda_0 \beta$.]

Note that this modified action is unchanged for plaquettes near the identity. This means that the

modification does not influence the formal continuum limit. Furthermore, perturbative aspects of the theory are completely unmodified. In particular, the scale factors for asymptotic freedom are independent of λ .

The action (9) and (10) with $\lambda = \infty$ was used in Ref. [9] ("positive plaquette model"). We study the phase structure of this model in the (β, λ) plane as well as the influence of small size lattice artifacts on "physical" values. From this point of view it is reasonable to use finite values of λ to understand better the structure of lattice theory.

It is perhaps instructive to study the role of lattice artifacts in two-dimensional ($D=2$) lattice gauge theory. Note first that, in two dimensions, because the coupling is dimensional (contrary to the $D=4$ case), the number of negative plaquettes at $\lambda=0$ dies out faster than exponentially with the lattice spacing; so, the density of Z_2 strings tends rapidly to zero when β tends to infinity. This gives us confidence that at high enough values of β negative plaquettes play a negligible role.

To study this further one can apply the character expansion

$$\begin{aligned} F(U) &\equiv \exp\left[\frac{1}{2}\beta \text{Tr} U + \lambda \text{sgn}(\text{Tr} U)\right] \\ &= \sum_{p=0; \frac{1}{2}; 1; \dots} d_p F_p(\beta; \lambda) \chi_p(U), \end{aligned} \quad (11)$$

where $\chi_p(U)$ is the character of p th representation of the $SU(2)$ group, $d_p = 2p + 1$ and

$$F_p(\beta; \lambda) = \frac{1}{d_p} \int dU F(U) \chi_p(U) = \frac{1}{2\pi d_p} \int_0^{4\pi} d\alpha \sin \frac{\alpha}{2} \sin \left[p + \frac{1}{2} \right] \alpha \exp \left[\beta \cos \frac{\alpha}{2} + \lambda \text{sgn} \left[\cos \frac{\alpha}{2} \right] \right]. \quad (12)$$

For an $N_t \times N_s$ lattice one can easily obtain the partition function Z in the limit $N_t, N_s \rightarrow \infty$:

$$Z = \exp[-(\beta + \lambda)N_{\square}] F_0^{N_{\square}}(\beta, \lambda), \quad N_{\square} = N_t N_s. \quad (13)$$

If $\lambda=0$ this reduces to the well-known expression for the partition function:

$$Z(\beta, \lambda=0) = \exp(-\beta N_{\square}) \left[\frac{2I_1(\beta)}{\beta} \right]^{N_{\square}}, \quad (14)$$

where $I_1(\beta)$ is the modified Bessel function.

Using Eqs. (11)–(13) one can easily produce an expression for the average plaquette $\langle \square \rangle$:

$$\langle \square \rangle(\beta; \lambda) \equiv 1 - \frac{1}{N_{\square}} \frac{\partial \ln Z}{\partial \beta} = \frac{F_{1/2}(\beta, \lambda)}{F_0(\beta, \lambda)}. \quad (15)$$

At $\beta=0$ $\langle \square \rangle(\beta=0; \lambda) = (4/3\pi) \tanh(\lambda)$. Figure 1(a) shows the ratio of plaquettes,

$$\gamma(\beta, \lambda) = \frac{\langle \square \rangle(\beta, \lambda)}{\langle \square \rangle(\beta, \lambda=0)}, \quad (16)$$

as a function of β at $\lambda = \infty$. At small β this ratio is high but in the weak-coupling regime ($\beta \rightarrow \infty$) $\gamma(\beta, \lambda = \infty) \rightarrow 1$. At very high values of β , negative plaquettes give a negligible contribution to the average plaquette, as it must be. But for Wilson loops the situation is different. The expression for the $l_1 \times l_2$ Wilson loop is of the form

$$\begin{aligned} W(l_1, l_2; \beta, \lambda) &= \left[\frac{F_{1/2}(\beta, \lambda)}{F_0(\beta, \lambda)} \right]^{l_1 l_2} \\ &\equiv [\langle \square \rangle(\beta, \lambda)]^{l_1 l_2} \end{aligned} \quad (17)$$

and

$$\begin{aligned} W(l_1, l_2; \beta, \lambda=0) &= \left[\frac{I_2(\beta)}{I_1(\beta)} \right]^{l_1 l_2} \\ &\equiv [\langle \square \rangle(\beta, \lambda=0)]^{l_1 l_2}. \end{aligned} \quad (18)$$

Therefore the ratio

$$\frac{W(l_1, l_2; \beta, \lambda)}{W(l_1, l_2; \beta, \lambda=0)} = [\gamma(\beta, \lambda)]^{l_1 l_2} \quad (19)$$

differs from unity more and more with increasing the size for the loop $l_1 l_2$; e.g., the "local" objects (negative plaquettes) can strongly influence large-scale objects.

Of course the strong dependence of Wilson loops on negative plaquettes does not mean that string tension has such a strong dependence also. Wilson loops, Eq. (17), have area-law behavior with the string tension σ depending on λ . At $\lambda=0$,

$$\sigma(\beta, \lambda=0) \sim \beta \ln \frac{I_1(\beta)}{I_2(\beta)} \rightarrow \frac{3}{2} \text{ at } \beta \rightarrow \infty. \quad (20)$$

Figure 1(b) represents the dependence of the string tension $\sigma \equiv \sigma(\beta, \lambda)$ on β at different values of λ . One can see that while the asymptotic behavior of σ does not depend on λ , in the intermediate region of β ($\beta \lesssim 8$) the dependence on the number of negative plaquettes is still present. This clearly illustrates how different actions with the same continuum limit can have differing finite lattice artifacts.

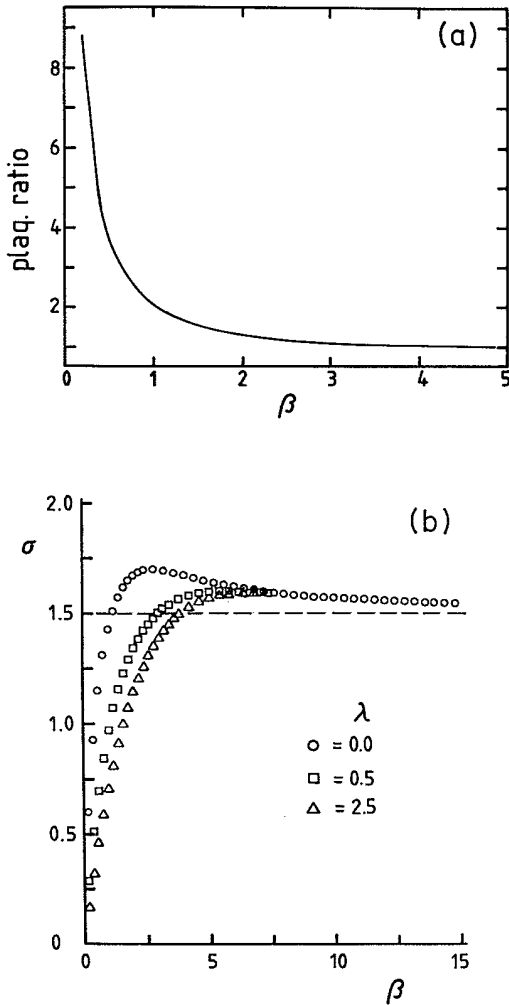


FIG. 1. The ratio of plaquettes $\langle \square \rangle(\beta, \lambda = \infty) / \langle \square \rangle(\beta, \lambda = 0)$ as a function of β in two-dimensional lattice gauge theory (LGT) (a); The dependence of $\sigma(\beta, \lambda)$ in $D=2$ LGT on β at different values of λ (b).

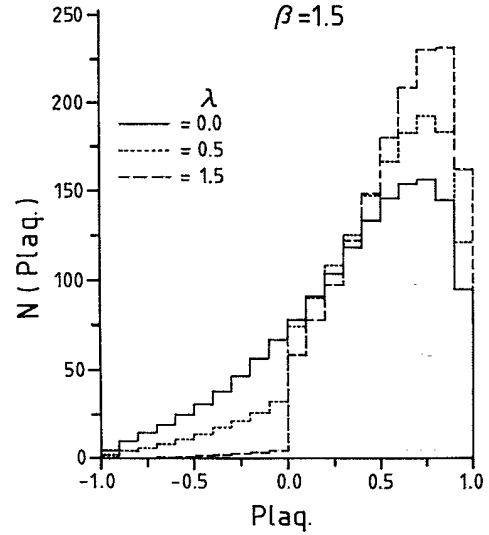


FIG. 2. Plaquette distribution at $\beta=1.5$ at different values of λ .

We now return to the four-dimensional SU(2) theory. Monte Carlo calculations were made on 4^4 , 6^4 , and 12^4 lattices with periodic boundary conditions. Some calculations were also made on the 1×6^3 lattice.

The modification of the action in Eqs. (9) and (10) changes, evidently, the distribution of plaquettes $P_\lambda(\square)$, suppressing (at $\lambda > 0$) negative values of \square . At nonzero values of λ the distribution develops a singularity at zero for the plaquette \square . The strength of this singularity increases with increasing $|\lambda|$. In Fig. 2 one can see the distributions $P_\lambda(\square)$ (normalized to the same value) at $\lambda=0, 0.5, 1.5$ and $\beta=1.5$. At negative values of λ an enhancement of negative plaquettes takes place.

Figure 3 represents the dependence of the number of

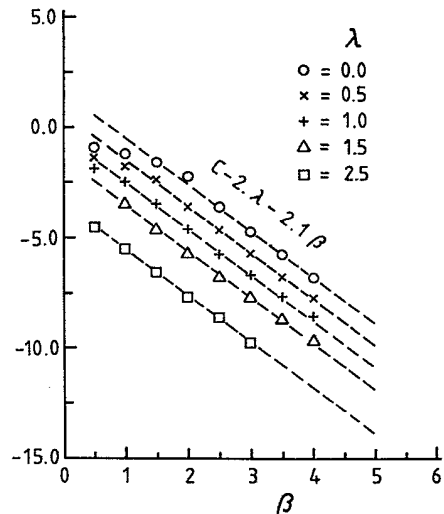


FIG. 3. The dependence of $\ln(N_- / N_0)$ on β at different values of λ .

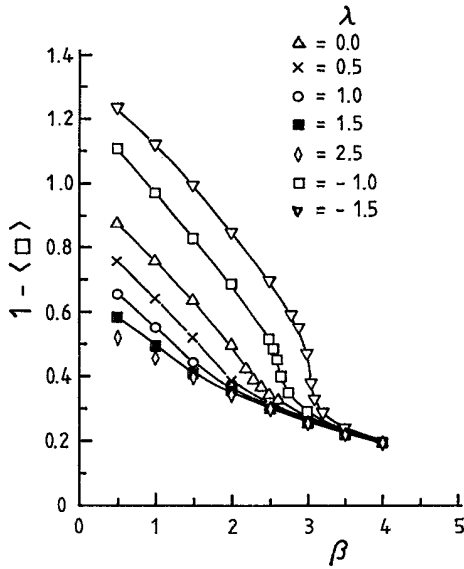


FIG. 4. The dependence of average plaquette $\langle \square \rangle$ on β at different values of λ .

negative plaquettes $\ln(N_-/N_\square)$ on β at different values of λ . Broken lines correspond to the exponential behavior

$$\sim \exp(-c_\beta \beta - c_\lambda \lambda), \tag{21}$$

where the coefficients c_β and c_λ are chosen to be $c_\beta = 2.0$ and $c_\lambda = 2.1$. Choosing $\lambda = \lambda_0 \beta$ one can obtain an exponential decay of the number of negative plaquettes in the continuum limit with any slope depending on the choice of λ_0 .

It is well known that in the theory with the standard Wilson action ($\lambda = 0$) there is a rapid change in behavior of the average plaquette $\langle \square \rangle(\beta)$ (crossover) at $\beta \sim 2.2$ [14,15]. At positive values of λ the dependence of

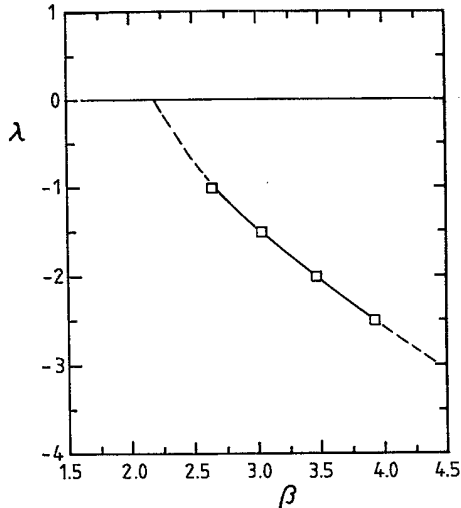


FIG. 5. Phase diagram in the (β, λ) plane.

$\langle \square \rangle(\beta)$ on β becomes increasingly smooth with increasing λ , and at large enough values of λ the crossover disappears (see Fig. 4). So, the boundary between the strong- and weak-coupling regions becomes invisible. At negative values of λ the crossover, to the contrary, becomes much more pronounced and at $\lambda \lesssim -1$ the first-order phase transition appears.

It is interesting to compare this behavior of $\langle \square \rangle(\beta)$ with that in the MP model [8] including the chemical potential λ_{MP} suppressing only Z_2 monopoles but not Z_2 strings. In this model at a positive and large enough λ_{MP} the crossover disappears but instead a first-order phase transition develops at $\beta \sim 1.0$ [10]. Behavior similar to ours takes place also in the monopoleless $SO(3)$ model [9].

In Fig. 5 we show the phase diagram of this model in the (β, λ) plane. At negative values of λ there is the line of first-order phase transitions along which the average values of the plaquette have a discontinuity. This line ends at $\lambda \sim -1$ and $\beta \sim 2.5$. Therefore the crossover in the lattice theory with the standard Wilson action ($\lambda = 0$) can be regarded as a shadow of this line in the (β, λ) plane.

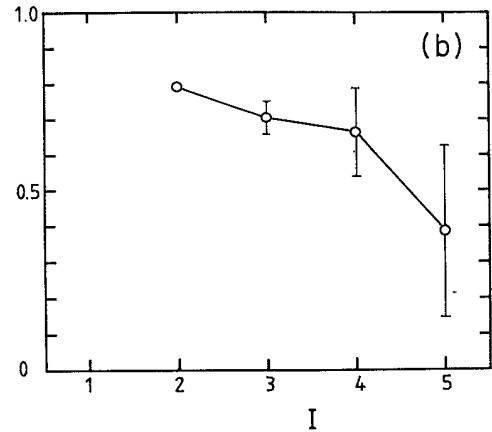
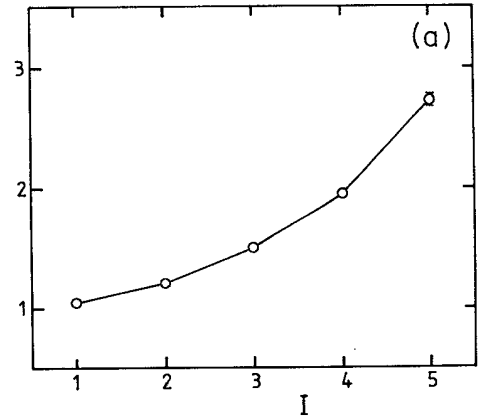


FIG. 6. The ratio of Wilson loops $W(I, I; \lambda = 0.5) / W(I, I; \lambda = 0)$ at $\beta = 2.5$ (a). The ratio $\chi(I; \lambda = 0.5) / \chi(I; \lambda = 0)$ at $\beta = 2.5$ (b). Data for $W(I, I; \lambda = 0)$ are from Ref. [16].

We calculated Wilson loops $W(I, J; \beta, \lambda)$ on the 12^4 lattice in the interval $2.0 \leq \beta \leq 2.5$. In Fig. 6(a) one can see the ratio of Wilson loops $W(I, I; \beta, \lambda=0.5)/W(I, I; \beta, \lambda=0)$ at $\beta=2.5$. Data for $W(I, I; \beta, \lambda=0)$ are from Ref. [16]. A comparison of Wilson loops in the modified theory with that for the standard Wilson theory shows the same behavior as in the case of two-dimensional theory: the more the size of the loop the more deviation from standard theory (and more influence of lattice artifacts). Note that at this value of β negative plaquettes make up less than 1%: $N_{\square^-}/N_{\square} \approx 0.01$.

A possibly more physical issue involves the behavior of the value $\chi(I; \beta, \lambda)$,

$$\chi(I; \beta, \lambda) = -\ln \frac{W(I+1, I+1; \beta, \lambda)W(I, I; \beta, \lambda)}{W(I+1, I; \beta, \lambda)W(I, I+1; \beta, \lambda)}, \quad (22)$$

which is connected with the string tension. In Fig. 6(b) one can see the behavior of the ratio $\chi(I; \lambda=0.5)/\chi(I; \lambda=0)$ at $\beta=2.5$. One can see the strong influence of lattice artifacts on string tension. This is indicative that the start of the scaling region can depend on the detailed action.

As is well known on an asymmetric lattice $N_t \times \infty^3$ a second-order deconfinement phase transition takes place at some critical value $\beta = \beta_c$ in standard SU(2) Wilson theory. The order parameter is the average Polyakov loop $\langle L \rangle$ which is defined as

$$\langle L \rangle \equiv Z^{-1} \int \prod_{\text{links}} dU_l L(\mathbf{x}) \exp[-S_W(U_l)], \quad (23)$$

where

$$L(\mathbf{x}) \equiv \frac{1}{2} \text{Tr} \left[\prod_{t=1}^{N_t} U_4(\mathbf{x}, t) \right]. \quad (24)$$

We expect $\langle L \rangle$ to be 0 for β below the critical point β_c and different from zero (in the infinite-volume limit) at $\beta > \beta_c$. On the finite lattice $N_t N_s^3$ the symmetry is never broken and $\langle L \rangle$ equals zero at all β because of tunneling between different minima of the effective potential. Nevertheless, the deconfinement phase on the infinite lattice manifests itself on the finite lattice with the double-peak distribution $P(\bar{L})$ where \bar{L} is the average of Polyakov loops through the whole lattice for a given configuration.

$$\begin{aligned} Z &= \int \prod_{\text{links}} dU_l \prod_{\square} \exp\left[\frac{1}{2}\beta \text{Tr} U_{\square} + \lambda \text{sgn}(\text{Tr} U_{\square})\right] \\ &= \int \prod_{\text{links}} dU_l \prod_{\square} \sum_p d_p F_p(\beta; \lambda) \chi_p(U_{\square}) \\ &= [F_0(\beta; \lambda)]^{N_{\square}} \int \prod_{\text{links}} dU_l \prod_{\square} [1 + 2A_{1/2}(\beta; \lambda) \chi_p(U_{\square}) + \dots], \end{aligned} \quad (25)$$

where

$$A_p(\beta; \lambda) = \frac{F_p(\beta; \lambda)}{F_0(\beta; \lambda)} \quad (26)$$

and coefficient functions $F_p(\beta; \lambda)$ are defined in (12).

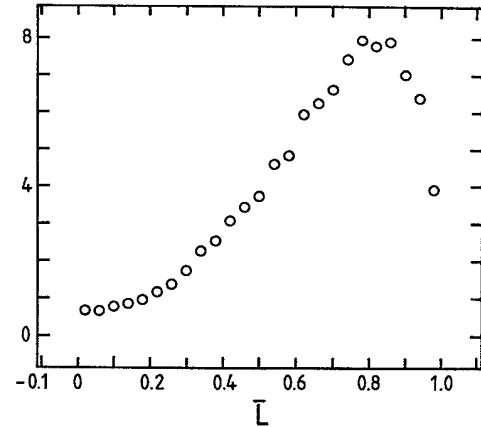


FIG. 7. Polyakov loop distribution for 1×6^3 lattice and $\beta=0.1$ at infinite lambda.

The strong influence of negative plaquettes on the position of the critical point β_c was observed in Ref. [9]. We would like to note here that in some extreme cases the abolishing of negative plaquettes can entail the disappearance of the finite-temperature phase transitions. We calculated the distribution of the Polyakov loop $P(\bar{L})$ on the lattice 1×6^3 at values of β in the interval $0.01 \lesssim \beta \lesssim 2.0$. On this lattice in the theory with standard Wilson action the critical point is at $\beta_c \approx 0.87$ [17]. In the modified theory without negative plaquettes ($\lambda = \infty$) the confinement phase does not exist. Figure 7 presents the distribution of $P(\bar{L})$ for $\beta=0.1$ and $\lambda = \infty$ on the lattice 1×6^3 . One can see the distribution has a sharply pronounced peak at $\bar{L} \sim 0.8$ and $\langle |\bar{L}| \rangle \approx 0.67$. The further decreasing of β down to $\beta=0.01$ does not change this picture. This means that whether or not the finite-temperature phase transition on the lattice with $N_t=1$ exists it is controlled by lattice artifacts.

To understand better the nature of this drastic influence of a rather small admixture of negative plaquettes on the thermodynamic properties of theory, let us represent the partition function Z in the form

Therefore all "observables" (Wilson loops, etc.) are defined by the set of coefficient functions $A_0, A_{1/2}, A_1, \dots$. Suppose for simplicity that only the first coefficient function $A_{1/2}(\beta, \lambda)$ plays a crucial role [$A_1(\beta, \lambda), \dots$, are essentially smaller]. Compare the be-

havior of $A_{1/2}(\beta, \lambda=0)$ (standard Wilson theory) with $A_{1/2}(\beta, \lambda \neq 0)$. Figure 8 shows the β dependence of $A_{1/2}(\beta, \lambda=\infty)$ (upper curve) and $A_{1/2}(\beta, \lambda=0)$ (lower curve). One can see that

$$A_{1/2}(\beta, \lambda=\infty) > A_{1/2}(\beta, \lambda=0)$$

at all β , and

$$A_{1/2}(\beta=0, \lambda=\infty) = \frac{4}{3\pi} \neq 0.$$

Choosing two values β_1 and β_2 in such a way that

$$A_{1/2}(\beta_1, \lambda=\infty) = A_{1/2}(\beta_2, \lambda=0)$$

one obtains $\beta_1 < \beta_2$. So the standard Wilson theory with $\beta = \beta_2 = 2.4$ corresponds (in the chosen approximation) to the modified theory with $\beta = \beta_1 \sim 1.0$. Note that because the renormalization scales for the two theories are the same, the size of this shift should decrease as we go to larger beta.

There can be other short-range fluctuations due to which the topological charge is not a well-defined object (at least, event by event) [6,18].

Another important point is that there can be lattice artifacts at other scales which are in addition to and independent of small size artifacts [8,12]. Indeed, if $\text{Tr}(U_{\square} U_{\square'})$ is less than zero, we can say that there is a string through the 2×1 rectangle. On the other hand, $\text{Tr}(U_{\square} U_{\square'}) \neq \text{Tr}(U_{\square}) \text{Tr}(U_{\square'})$ and this means that the presence of this "thick" string does not depend on the presence of "thin" strings through the plaquettes \square and \square' (see, e.g., [10]). Therefore at all scales the Z_2 artifacts can exist. But the study of those larger size artifacts is beyond the scope of this paper.

In an optimistic scenario the λ modification of the action may improve the approach to scaling. On the other hand, it may be that the negative plaquettes in the Wilson

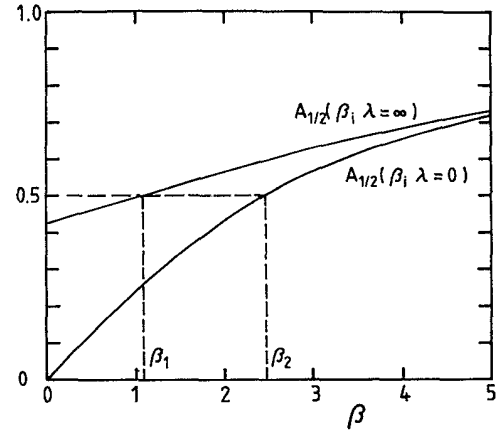


FIG. 8. The dependence of the coefficient functions $A_{1/2}(\beta, \lambda)$ on β at $\lambda = \infty$ (upper curve) and at $\lambda = 0$ (lower curve).

theory are accidentally canceling some other artifact, making scaling fortuitously good in the standard approach. At the same time it is impossible to exclude the situation that this λ -modified model belongs to some other universality class than the Wilson theory. To answer this question it is necessary to calculate the "physical" values, e.g., the ratio of the string tension to the critical temperature $\sigma^{1/2}/\theta_c$ and compare them with that calculated in the standard Wilson theory. This work is in progress.

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