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Quark bags and local field theory. II. Confinement of Fermi and vector fields

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We extend earlier work on obtaining the "bag" model of Chodos, Jaffe, Johnson, Thorn, and Weisskopf as a limit of a local field theory. As before, the discussion is on a classical level. We show how to confine Fermi and vector fields into the bag solutions found previously. In an appropriate limit we obtain exactly the Chodos *et al.* model for confinement of all fields except non-Abelian gauge fields. For the latter fields we argue that it is impossible from a local theory to obtain the same boundary conditions used in the Chodos *et al.* model. The boundary conditions we find do not have the virtue of completely eliminating "color" nonsinglet states from the theory. We propose an alternative scheme based on a single triplet of quarks and an Abelian gauge field. The baryons are bound states of three quarks and one additional scalar constituent.

One of us[1] recently showed how one version of the "bag" model of Chodos, Jaffe, Johnson, Thorn, and Weisskopf[2] for extended hadrons can be obtained as a limit of a local field theory. In this limit the quarks are confined in a finite spatial region referred to as the bag. For simplicity, Ref. 1 used a scalar field to represent the quarks.

In this paper we generalize the previous result to include confinement of Fermi and vector fields in the bag. As in Ref. 1, our discussion involves only classical field theory. We find that the MIT model for confining Fermi fields and vector fields of an Abelian gauge theory can be obtained as a limit of a local field theory. Although we can confine non-Abelian gauge fields as well, we find that from a local theory it is impossible to obtain the same boundary conditions used in the MIT model. The boundary conditions we obtain for the non-Abelian theories do not eliminate color nonsinglet hadron states, although such states may be heavy and hard to produce.[3] Because of the problems with non-Abelian theory, we present a simple confinement scheme using an Abelian gauge meson, a single triplet of quarks, and one additional scalar constituent for baryons.

The fermion confinement is particularly interesting because it provides an example which contrasts with the model originally presented by Vinciarelli[4] and extensively discussed by Bardeen, Chanowitz, Drell, Weinstein, and Yan.[5] In the latter picture a fermion quark is confined to a thin shell on the surface of the bag, while in our model a quark is free and has a small mass inside the bag.

I. SPINOR CONFINEMENT

We begin with the Lagrangian density

$$\mathcal{L} = \tfrac{1}{2}\partial_\mu\phi\partial_\mu\phi - V(\phi) + \bar{\psi}i\partial\!\!\!/\psi + \lambda(\phi-\beta)\bar{\psi}\psi - m\bar{\psi}\psi , \tag{1}$$

where ψ is the spinor field which is to be confined, ϕ is a real scalar field, and $V(\phi)$ is the potential given in Ref. 1:

$$V(\phi) = \alpha\left(\frac{\phi^4}{4} - (\beta+\gamma)\frac{\phi^3}{3} + \beta\gamma\frac{\phi^2}{2}\right), \tag{2}$$

with the constraint

$$\frac{1}{2} \leq \frac{\gamma}{\beta} < 1 . \tag{3}$$

The quantities α, β, γ, and m are positive parameters. The parameter m will be the effective quark mass inside the bag. The equations of motion for the fields are

$$\Box\phi = -\alpha\phi(\phi-\gamma)(\phi-\beta) + \lambda\bar{\psi}\psi , \tag{4a}$$

$$i\partial\!\!\!/\psi = \lambda(\beta-\phi)\psi + m\psi . \tag{4b}$$

As discussed in Ref. 1, a bag is a finite region of space in which the field ϕ is approximately β. This region is surrounded by the vacuum with ϕ near zero. The parameters will be adjusted so that the transition of ϕ from β to zero takes place in a thin "skin" enclosing the bag. The fermion quark field is trapped inside this bag. In the absence of ψ the state of lowest energy has $\phi(x) = 0$ for all x. However, in the presence of quarks the bag solution has lower energy when the effective mass, $\lambda\beta$, of the quark outside the bag is sufficiently large.

In order to have the model of Ref. 2 with Dirichlet boundary conditions, we need to show that the above local field theory has a limit in which the action for a bag solution is given by

$$W=\int_V d^4x(\bar{\psi}i\partial\!\!\!/\psi-m\bar{\psi}\psi-B)+\int_{\bar{V}} d^4x(\bar{\psi}i\partial\!\!\!/\psi-M\bar{\psi}\psi), \tag{5}$$

where V is the region of space-time swept out by the interior of the bag, and $\bar{V}$ is the complement of V. The parameter M is the quark mass outside the bag and is to be taken to infinity.

Now we consider the required conditions to obtain the action (5) from the Lagrangian (4). The effective masses of ψ and ϕ outside the bag should be large; this yields

$$m_{\psi,E}=M=\lambda\beta+m\to\infty, \tag{6}$$

$$m_{\phi,E}{}^2=\alpha\beta\gamma\to\infty. \tag{7}$$

To obtain the term $-\int_V d^4x\,B$ in the expression for the action, we require

$$V(\beta)=\frac{1}{6}\,\alpha\beta\left(\frac{\gamma}{\beta}-\frac{1}{2}\right)=B. \tag{8}$$

The vanishing of the thickness and energy of the bag skin was shown in Ref. 1 to require

$$\Delta^{-2}\sim\alpha\beta^2\to\infty, \tag{9}$$

$$E_S{}^2\sim\alpha\beta^6\to 0, \tag{10}$$

where Δ and E_S are, respectively, the thickness and energy per unit area of the bag skin. Note that these conditions require $\beta\to 0$, i.e., a small shift in the field ϕ produces the bag. We will show below that the ψ field does not substantially affect conditions (9) and (10).

Inside the bag we analyze the field behavior a little more explicitly than in the scalar case of Ref. 1. Introduce the field $\epsilon(x)$ by the equation

$$\phi(x)=\beta+\epsilon(x). \tag{11}$$

Then Eq. (4) becomes

$$\Box\epsilon(x)+\alpha\beta^2(1-\gamma/\beta)\epsilon(x)=\lambda\bar{\psi}(x)\psi(x)+O(\epsilon^2(x)), \tag{12a}$$

$$(i\partial\!\!\!/-m)\psi(x)=\lambda\epsilon(x)\psi(x). \tag{12b}$$

We require the effective ϵ mass inside the bag to be very large, i.e.,

$$m_{\epsilon,I}{}^2=\alpha\beta^2(1-\gamma/\beta)\to\infty. \tag{13}$$

Since we are looking for a bag solution where $\epsilon(x)$ is not rapidly varying inside the bag, we ask that

$$|\Box\epsilon(x)|\ll\alpha\beta^2(1-\gamma/\beta)|\epsilon(x)|, \tag{14}$$

implying from Eq. (12a)

$$\epsilon(x)\approx\frac{\lambda}{\alpha\beta^2(1-\gamma/\beta)}\bar{\psi}\psi. \tag{15}$$

Thus the quark field induces a shift in $\epsilon(x)$ from zero. We require that this shift give an insignificant change to $V(\phi)$

$$V(\beta-\epsilon)-V(\beta)\approx m_{\epsilon,I}{}^2\epsilon^2\sim\frac{\lambda^2(\bar{\psi}\psi)^2}{\alpha\beta^2(1-\gamma/\beta)}\to 0. \tag{16}$$

Since $\bar{\psi}\psi$ is expected to remain finite in the bag, and we will see later that $\gamma\sim\frac{1}{2}\beta$, this condition reduces to

$$\frac{\lambda^2}{\alpha\beta^2}\to 0. \tag{17}$$

In order to have essentially free quarks of mass m inside the bag, we require that the interaction term $\lambda\epsilon(x)\psi(x)$ in Eq. (12b) be small. This term represents the possibility of quark-quark interactions via the exchange of ϵ mesons. Thus we require that

$$\frac{\lambda^2}{m_{\epsilon,I}{}^2}\sim\frac{\lambda^2}{\alpha\beta^2}\to 0. \tag{18}$$

This requirement is identical to Eq. (17).

We still must show that the ψ field does not significantly alter the skin properties. Thus in the skin we must have

$$\frac{|\lambda(\phi-\beta)\bar{\psi}\psi|\Delta}{E_S}\sim\frac{\lambda}{\alpha\beta^3}\to 0. \tag{19}$$

This follows from conditions (6) and (17):

$$\frac{\lambda}{\alpha\beta^3}=\frac{\lambda^2}{\alpha\beta^2}\times\frac{1}{\lambda\beta}\to 0. \tag{20}$$

Finally we note that

$$\Delta^2 m_{\psi,E}{}^2\sim\frac{\lambda}{\alpha\beta}=\frac{\lambda^2}{\alpha\beta^2}\times\frac{1}{\lambda\beta}\times\beta^2\to 0, \tag{21}$$

by conditions (6), (18), and $\beta\to 0$. This means that for excitations of the quark field with energies small or comparable to the external quark mass, the skin thickness is unimportant to the behavior of the ψ field. This ensures that we will obtain the Dirichlet boundary conditions for the quark field at the bag skin.

To show that all the above conditions are mutually consistent, we note that they are met by the parametric representation

$$\alpha=R_1\beta^{-(4+p_1)}, \tag{22a}$$

$$\lambda^2=R_2\beta^{-(2+p_2)}, \tag{22b}$$

$$0<p_2<p_1<2, \tag{22c}$$

$$\gamma=\frac{\beta}{2}\left(1+\frac{12B}{R_1}\beta^{p_1}\right), \tag{22d}$$

where R_1 and R_2 are positive constants and $\beta \to 0$.

In the above limit the local field theory given by the Lagrangian (1) describes the absolute fermion quark confinement of the bag in Ref. 1. If β is small but not zero, this local theory gives a practical confinement of quarks similar in philosophy to that of Refs. 4 and 5. Indeed, those papers use a Lagrangian given by a different choice of the parameters in Eq. (1), i.e.,

$$\frac{\gamma}{\beta} = \frac{1}{2}, \tag{23a}$$

$$m = \lambda \frac{\beta}{2}. \tag{23b}$$

Equation (23a) implies $B=0$; the skin energy holds the bag together. The inside and outside of the resulting bag are locally indistinguishable, and the quarks are light only on the bag surface.

II. VECTOR CONFINEMENT

We use a gauge theory to introduce vector mesons. These mesons should acquire a large mass outside the bag in order for them to be confined in the bag. Staying within the gauge theory framework, we use the Higgs mechanism[6] to provide this large mass.[7] We choose the parameters of the theory so that the gauge symmetry is spontaneously broken outside the bag, providing the vector mesons with a mass; on the other hand, inside the bag the gauge symmetry is preserved, and the gauge mesons remain massless.

To illustrate the procedure, consider a single gauge meson with a corresponding U(1) gauge group. We study the Lagrangian

$$\begin{aligned}\mathcal{L} = &\tfrac{1}{2}\partial_\mu\phi\partial_\mu\phi - V(\phi) - \tfrac{1}{4}F_{\mu\nu}F_{\mu\nu} \\ &+ (\partial_\mu + igA_\mu)\chi^*(\partial_\mu - igA_\mu)\chi \\ &+ \rho_1(\chi^*\chi)(\tfrac{1}{2}\beta - \phi) - \rho_2(\chi^*\chi)^2 \\ &+ i\bar\psi(\partial\!\!\!/ - ig A\!\!\!/)\psi + \lambda(\phi-\beta)\bar\psi\psi - m\bar\psi\psi,\end{aligned} \tag{24}$$

where $V(\phi)$ is the potential of Eq. (2), A_μ is the vector field, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, χ is a complex scalar field, ρ_1 and ρ_2 are constants, and g is the coupling constant of the gauge theory. Inside the bag, where $\phi \approx \beta$, χ has positive mass squared

$$m_{\chi,I}^2 = \tfrac{1}{2}\rho_1\beta, \tag{25}$$

and A_μ describes massless vector mesons. However, outside the bag, when $\phi \approx 0$, the mass squared of the χ field is negative, $m_{\chi,E}^2 = -\frac{1}{2}\rho_1\beta$, and spontaneous breakdown of the gauge theory will occur in the standard manner.[7] The theory then possesses a massive vector field $V_\mu(x)$ of mass

$$m_{V,E}^2 = \frac{g^2\rho_1\beta}{2\rho_2} \tag{26}$$

and a massive real scalar field $\rho(x)$ of mass

$$m_{\rho,E}^2 = \rho_1\beta.$$

The spontaneous symmetry breaking also contributes an amount $\rho_1^2\beta^2/16\rho_2$ to the pressure B holding the bag together. Thus, in Eq. (22d) determining γ, B should be replaced with $B - \frac{1}{16}(\rho_1\beta)^2/\rho_2$. As before, one can take an appropriate limit of the parameters to make all masses large except for those of the quarks and vector mesons in the bag. However, we must still consider the boundary conditions for $F_{\mu\nu}(x)$ at the bag skin.

Our discussion of the boundary conditions is motivated by the observation of Nielsen and Olesen[8] that the Higgs Lagrangian describes a relativistic generalization of the Landau-Ginzburg[9] phenomenological theory of superconductivity. Thus our bag can be visualized as an insulating bubble inside a superconducting vacuum. Borrowing terminology from electrodynamics, we consider the electric and magnetic fields associated with the field $F_{\mu\nu}$

$$\begin{aligned}(\vec{E})_i &= -F_{0i} = \left(-\frac{\partial\vec{A}}{\partial t} - \vec\nabla A_0\right)_i, \\ (\vec{B})_i &= -\epsilon_{ijk}F_{jk} = (\vec\nabla\times\vec{A})_i.\end{aligned} \tag{28}$$

We now go to the Lorentz frame of a point on the bag surface. Equation (28) implies $\vec\nabla\cdot\vec{B}=0$ everywhere, i.e., magnetic flux is conserved. Outside the bag the gauge field has a large mass and is thus rapidly attenuated at energies below the vector-meson production threshold. Applying Gauss's law for magnetic fields to a small region enclosing a portion of the bag surface shows that magnetic flux cannot escape the bag and thus at the boundary $\vec{B}$ is parallel to the surface. Equation (28) also implies $\vec\nabla\times\vec{E} = -\partial\vec{B}/\partial t$. Applying Stokes's theorem near the surface shows that $\vec{E}$ at the bag surface must be normal. These boundary conditions are those at the surface of a superconductor and can be written

$$\epsilon_{\mu\nu\rho\sigma}n_\nu F_{\rho\sigma} = 0 \tag{29}$$

on the bag boundary. Here $\epsilon_{\mu\nu\rho\sigma}$ is the totally antisymmetric tensor with $\epsilon_{0123}=1$, and n_μ is the four-vector inward normal to the bag surface. Equation (29) is in sharp contrast to the boundary conditions used in Ref. 2, where

$$n_\mu F_{\mu\nu} = 0 \tag{30}$$

on the boundary. Equation (30) gives a parallel electric field and a normal magnetic field at the surface. In Ref. 2, the parallel electric field at the surface is essential to the use of Gauss's law in eliminating bags containing a single quark.

The nonlinear boundary condition representing the contribution of the fields $F_{\mu\nu}$ to the pressure balancing the external pressure B is also modified in this picture. We find that this contribution to the outward pressure is $\frac{1}{4}F_{\mu\nu}F_{\mu\nu}=\frac{1}{2}(\vec{B}^2-\vec{E}^2)$, differing in sign from the model of Ref. 2. The full boundary conditions are

$$B=\tfrac{1}{4}F_{\mu\nu}F_{\mu\nu}+\tfrac{1}{2}n_\mu\,\partial_\mu(\bar{\psi}\psi)$$
$$=\tfrac{1}{2}\vec{B}^2-\tfrac{1}{2}\vec{E}^2+\tfrac{1}{2}n_\mu\,\partial_\mu(\bar{\psi}\psi), \tag{31a}$$

$$-i\not{n}\psi=\psi, \tag{31b}$$

$$\epsilon_{\mu\nu\rho\sigma}n_\nu F_{\rho\sigma}=0\,. \tag{31c}$$

The negative sign of the contribution of the electric field to the outward pressure can be understood as the attraction of the electric field for the charge density induced in the bag skin.

The authors of Ref. 2 have emphasized that consistent boundary conditions must yield a conserved energy and momentum. As we started with a translationally invariant local field theory, energy-momentum conservation is implicit. The states we consider are spatially bounded; thus, the limits discussed above cannot affect this conservation. An exercise in tensor manipulation confirms this by showing that the stress-energy tensor inside the bag

$$T_{\mu\nu}=F_{\mu\lambda}F_{\lambda\nu}+\tfrac{1}{4}g_{\mu\nu}F_{\rho\sigma}F_{\rho\sigma}+\tfrac{1}{2}i\bar{\psi}\gamma_\mu(\partial_\nu-ieA_\nu)\psi$$
$$-\tfrac{1}{2}i[(\partial_\nu+ieA_\nu)\bar{\psi}]\gamma_\mu\psi+g_{\mu\nu}B \tag{32}$$

satisfies

$$n_\mu T_{\mu\nu}=0 \tag{33}$$

on the bag surface by virtue of the boundary conditions (31). As noted in Ref. 2, Eq. (33) guarantees four-momentum conservation.

The boundary conditions of Eq. (31) suffer the serious shortcoming that they do not allow the use of Gauss's theorem to rule out bags containing a net charge. However, the roles of electric and magnetic fields are merely interchanged in these boundary conditions as compared to those of Ref. 2. This suggests that to obtain a model equivalent to that of Ref. 2, we should allow the quarks to be magnetic rather than electric sources, i.e., the quarks should be magnetic monopoles with respect to the vector field.[10] (This, of course, has nothing to do with their behavior with respect to physical electromagnetic fields.) This prescription for Abelian gauge theories gives the model of Ref. 2 as a limit of a local theory.[11] However, for non-Abelian theories *this does not work*. The required analog of a magnetic monopole does not exist in a non-Abelian gauge theory.

In a non-Abelian gauge theory the symmetry between electric and magnetic fields in the absence of external sources is lost because the vector mesons themselves carry electric, and not magnetic, charge. To see the difficulty, note that we need a multiplet of magnetic monopoles and a corresponding conserved magnetic current $j_\mu^{M,\alpha}(x)$ coupled to the vector mesons via

$$\partial_\mu F^\alpha_{\nu\rho}(x)\epsilon_{\mu\nu\rho\sigma}=j_\sigma^{M,\alpha}(x)\,. \tag{34}$$

Here α is the internal-symmetry index associated with the non-Abelian gauge group. The magnetic current has nonvanishing matrix elements between monopole states

$$\langle p',M_i|j_\mu^{M,\alpha}(0)|p,M_j\rangle\neq 0 \text{ for some } i,j\ , \tag{35}$$

where $|p,M_j\rangle$ is a single-particle state of the monopole M_j with momentum p and where j is an index labeling the monopoles in the multiplet. Since the fields $F^\alpha_{\mu\nu}$ possess electric charge, Eq. (34) implies that the $j_\sigma^{M,\alpha}$ also carry electric charge. As charge is conserved, Eq. (35) indicates that the monopoles cannot all be electrically neutral. Furthermore, because of the non-Abelian nature of the group, the expression (35) cannot vanish for some $i\neq j$. This means that $j_\mu^{M,\alpha}(x)$ can change magnetic charge and thus the vector mesons must carry magnetic charge. The only consistent solution is for all particles to have the same ratio of electric to magnetic charge. A redefinition of the fields converts such a theory into one with only electric charges.

The requirement of Ref. 2 that the magnetic field be normal to the bag surface renders the surface a magnetic source. Any corresponding local theory must contain particles which are magnetic sources, an impossibility in non-Abelian theories. Consequently, the bag model of Ref. 2 containing non-Abelian colored gauge mesons cannot be obtained as a limit of a local theory.

We remark that the authors of Ref. 2 derive the boundary conditions for vector fields in a rather different manner than used for scalar or spinor fields. The quark-field boundary conditions were obtained by giving the quark a large mass outside the bag and at the end taking this mass to infinity. Such a procedure follows naturally in a local field-theoretical derivation of the bag. However, for vector fields the boundary conditions are obtained by absolutely excluding the fields from the exterior of the bag. This procedure gives different conditions from those that follow from giving the fields a large mass outside the bag, as occurs in our local theory. For Abelian theories we can circumvent this difficulty through use of the theory of magnetic monopoles. This cannot be done with non-Abelian gauge theories.

In the framework of Abelian gauge theories one can obtain a viable model for quark confinement using a single triplet of quarks. These quarks are all given the same value of magnetic charge with respect to an Abelian gauge field confined in the bag as discussed above. As usual, the mesons consist of a quark and an antiquark confined in a bag. In order to obtain baryons containing three quarks, we also confine a scalar field carrying a magnetic charge of minus three times that of the quarks. We refer to the quanta of this field as core mesons. The baryons are constructed of three quarks and one core meson, all confined in our local bag. Although we introduce the core meson by hand, it might also be generated out of a spontaneously broken non-Abelian gauge theory in the manner recently discussed by Mandelstam.[12] The quarks should be parafermions of order 3 in order to have statistics consistent with the observed hadron spectrum.[13] This scheme gives an absolute confinement of quarks in baglike structures arising from a local field theory.

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