Quantum fluctuations and the bag model*

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We consider a path-integral prescription for quantizing a variation of the bag model of Chodos *et al.* We treat the bag shape as an independent dynamical variable where associated kinetic terms in the Lagrangian have been taken to zero. We solve exactly a version of the model where a scalar field has a different mass inside and outside the bag. This solution is a free-field theory of a particle with a mass that is the root mean square of the input interior and exterior masses. No remnants of the semiclassical bag structures remain. The fact that the quantum fluctuations have so strongly altered the structure of the theory is associated with the lack of any structure in the surface of the bag. We argue that a surface tension alone is inadequate to control these fluctuations; both a surface tension and a surface thickness are needed.

I. INTRODUCTION

The bag model for quark confinement represents an attempt to reconcile the successes of the quark model with the nonobservation of free quarks.[1] By approximating the bag with a classical spherical cavity in which quarks are confined, the model makes several strikingly accurate predictions on hadronic properties.[2] From one point of view, these successes suggest that the model provides a good phenomenological approximation to some not yet understood confinement mechanism, perhaps the oft-advertised Yang-Mills gauge theory.[3] In previous work we have shown on a classical level how most features of the bag model followed in a certain limit of a conventional local relativistic field theory.[4] The achievements of the bag model encourage further study of extended objects in field theory as a possible first approximation to hadronic physics.[5]

From another point of view, the starting Lagrangian of the bag model could possess a deeper significance than purely phenomenological. The theory may represent a new type of fundamental field theory with relevance to the strong interactions. This paper investigates this second viewpoint. Using a Feynman path-integral prescription,[6] we attempt to define a quantum theory for the bag. We perform a sum over all possible bag configurations, with no restriction on the shapes or number of virtual bags present at any time. By integrating over bag shapes independently of the confined fields, we are treating the bag shape as an independent variable. Such a procedure is suggested if we consider the bag as arising in a limit of a conventional field theory or if we wish to introduce a surface tension. However, we must emphasize that this is not an equivalent procedure to determining the classical bag shape as a function of the fields and then evaluating the path integral over the fields alone. Thus our results may not follow from other quantization procedures.

To define the integral over bag shapes we work on a hypercubical lattice and at the end take the lattice spacing to zero. This procedure indicates that severe quantum fluctuations destroy the bag-like structures of the theory. In their place we find a local theory where the fields have properties that are the average of those inside and outside the bag. This result is rather surprising in that it implies that our quantization prescription does not have a classical limit. This is contrary to the usual notion that the path-integral method naturally gives the classical theory as $\hbar$ is taken to zero. We suspect that the problem is associated with the lack of any surface energy and thickness in the original formulation. Inclusion of such properties should provide an additional damping to the path integral over bag shapes. Recently a version of the model with surface tension but no surface thickness has been presented.[7] We investigate this possibility and find that such a term is insufficient to control the fluctuations mentioned above. The local theory of Ref. 4 gives the bags both surface energy and thickness before a limit giving the classical bag of Ref. 1 is taken. It has been argued that the quantized version of this theory will possess particle states corresponding to the classical bag-like structures.[8]

Shalloway and Rebbi have discussed quantum corrections to the bag theory by quantizing small fluctuations around classical bag solutions.[9] Rebbi has also considered a path-integral approach to the bag in two dimensions.[10] All these papers ignore those fluctuations in which virtual bags appear in the vacuum and virtual regions of normal vacuum appear inside the bag. It is precisely these fluctuations that are so severe. Note that such fluctuations are also ignored in the formal solution to the bag in two-dimensional space-time.[1]

The outline of this paper is as follows. In Sec. II we formulate and evaluate the path integral over bag configurations in a theory with a scalar field possessing a different mass inside and outside the bag. We find the resulting theory is that of free particles with a mass squared that is the average input mass squared. In Sec. III we perform the path integrations in another order to show that the result is unchanged. In Sec. IV we argue that the addition of a surface tension does not affect our conclusions. In Sec. V we conclude with some general remarks.

II. BAGS AND PATH INTEGRALS

The bag model begins by giving a field different properties inside and outside a region of space called the bag. For simplicity we study a single scalar field and consider the action

$$L(t)=\int_V d^3x\{\tfrac{1}{2}[\partial_\mu\phi(x)]^2-\tfrac{1}{2}m^2[\phi(x)]^2-B\}$$

$$+\int_{\overline{V}} d^3x\{\tfrac{1}{2}[\partial_\mu\phi(x)]^2-\tfrac{1}{2}M^2[\phi(x)]^2\}. \qquad (2.1)$$

Here V is the region of space called the bag, $\overline{V}$ its complement, m and M the respective masses of the field ϕ inside and outside the bag, and the "bag constant" B represents an external pressure acting to compress the bag. In Ref. 1 this Lagrangian is discussed in the limit $M\to\infty$, which classically confines the field entirely inside the bag. We will find this limit is singular in the quantum theory and thus we keep M finite.

Note that the Lagrangian in Eq. (2.1) has no explicit dependence on time derivatives of V. This leaves open two interpretations for its dynamical character in a quantum theory. We can consider this Lagrangian as representing a limit of some theory with nonvanishing kinetic term for V, such as the bag model with surface tension as discussed in Sec. IV or a local field-theoretical bag as studied in Ref. 4. This is the point of view adopted in this paper, with the consequence that in the path-integral quantization we sum over volumes and fields independently.

A second interpretation of V is to specify it as a given function of the field ϕ before quantization. Then the field ϕ is considered as the only independent variable to be integrated over for quantization. The specification used in Ref. 1 is that V maximizes the Lagrangian in Eq. (2.1) for a given field configuration $\phi(x)$. This amounts to

$$V(\phi)=\{\vec{x}\,|\,\tfrac{1}{2}(M^2-m^2)\phi^2(\vec{x},t)-B\geq 0\},$$

eliminating V in terms of ϕ gives a nonpolynomial Lagrangian that has been discussed by Chodos and by Kazama and Goldhaber.[11] It is presumably not equivalent on a quantum level to the theory we consider here.

In Eq. (2.1) we have given the same quadratic kinetic term to $\phi(x)$ both inside and outside the bag. This means there is no direct coupling between the volume V and time derivatives of ϕ. We have done this to avoid possible complications in the path-integral formalism which arise with derivative couplings. The explicit surface tension introduced in Sec. IV will also be a quadratic form in the time derivatives of the variables describing the bag shape. With purely quadratic kinetic terms, the usual path-integral prescription is to sum the exponential of i times the action over all independent dynamical variables. Thus we are led to study the quantity

$$W(J)=N\int d\phi\sum_V \exp\left[i\left(\mathcal{S}+\int d^4x\,J(x)\phi(x)\right)\right], \qquad (2.2)$$

where $\mathcal{S}$ is the action

$$\mathcal{S}=\int_{-\infty}^{\infty}dt\,L(t). \qquad (2.3)$$

Here the expression $\int d\phi\sum_V$ represents a sum over all bag and field configurations in a way which we will define by placing the theory on a discrete space-time lattice. The structure of the theory is probed by the external source $J(x)$ which for simplicity we couple linearly to $\phi(x)$. In a theory of non-Abelian gauge mesons coupled to quarks, this source should be coupled to some gauge singlet operator such as the electromagnetic current. The normalization factor N is chosen such that

$$W(0)=1. \qquad (2.4)$$

The postulate of path-integral quantization is that $W(J)$ generates Green's functions via the formula

$$\langle 0|T(\phi(x_1)\cdots\phi(x_n))|0\rangle=\left(\frac{-i\partial}{\partial J(x_1)}\right)\cdots\left(\frac{-i\partial}{\partial J(x_n)}\right)W(J)\bigg|_{J=0} \qquad (2.5)$$

where $|0\rangle$ is the ground state of the theory.

As a first step in defining the sum over volumes, we introduce a variable $S(x)$ defined by

$$S(x)=\begin{cases}+1, & x\in V\\ -1, & x\in\overline{V}.\end{cases} \qquad (2.6)$$

With this definition we rewrite Eq. (2.2) as

$$W(J)=N\int d\phi\exp\left[i\int d^4x\left(\tfrac{1}{2}(\partial_\mu\phi)^2-\frac{1}{2}\left(\frac{M^2+m^2}{2}\right)\phi^2+J\phi-\tfrac{1}{2}B\right)\right]\sum_V\exp\left[i\int d^4x\,S(x)[\tfrac{1}{4}(M^2-m^2)\phi^2-\tfrac{1}{2}B]\right]. \tag{2.7}$$

The definition of the sum over V is now formulated by dividing spacetime into a hypercubical lattice of spacing a. We then require that each fundamental hypercube of the lattice is either entirely inside or outside of V. Labeling the basic hypercubes by an index i, we require that $S(x)$ assume the same value S_i throughout the ith hypercube. Thus we write

$$\begin{aligned}W(J)&=N\int d\phi\exp\left[i\int d^4x\left(\tfrac{1}{2}(\partial_\mu\phi)^2-\frac{1}{2}\left(\frac{m^2+M^2}{2}\right)\phi^2+J\phi-\tfrac{1}{2}B\right)\right]\prod_i\left\{\sum_{S_i=\pm1}\exp\left[iS_i\int_i d^4x[\tfrac{1}{4}(M^2-m^2)\phi^2-\tfrac{1}{2}B]\right]\right\}\\&=N\int d\phi\exp\left[i\int d^4x\left(\tfrac{1}{2}(\partial_\mu\phi)^2-\frac{1}{2}\left(\frac{m^2+M^2}{2}\right)\phi^2+J\phi-\tfrac{1}{2}B\right)\right]\exp\left[\sum_i\ln\left(2\cos\left(\int_i d^4x[\tfrac{1}{4}(M^2-m^2)\phi^2-\tfrac{1}{2}B]\right)\right)\right].\end{aligned} \tag{2.8}$$

Here $\int_i d^4x$ denotes integration over the ith subvolume. In the limit of small lattice spacing we expand

$$\begin{aligned}\sum_i\left[\ln\left(2\cos\left(\int_i d^4x[\tfrac{1}{4}(M^2-m^2)\phi^2-\tfrac{1}{2}B]\right)\right)\right]&=\sum_i\ln2-\tfrac{1}{2}\sum_i\left(\int_i d^4x[\tfrac{1}{4}(M^2-m^2)\phi^2-\tfrac{1}{2}B]\right)^2+\cdots\\&=\frac{\Omega}{a^4}\ln2-\tfrac{1}{2}a^4\int d^4x[\tfrac{1}{4}(M^2-m^2)\phi^2(x)-\tfrac{1}{2}B]^2+O(a^6).\end{aligned} \tag{2.9}$$

Here Ω is the volume of space, and we have kept the term of order a^4 for later discussion. Dropping terms that vanish as $a\to0$, we find

$$W(J)=N'\int d\phi\exp\left[i\int d^4x\left(\tfrac{1}{2}(\partial_\mu\phi)^2-\frac{1}{2}\left(\frac{m^2+M^2}{2}\right)\phi^2-J\phi\right)\right], \tag{2.10}$$

where we have absorbed some divergent factors in N',

$$N'=N\exp\left(\frac{\Omega}{a^4}\ln2-\tfrac{1}{2}i\Omega B\right). \tag{2.11}$$

Equation (2.10) is the usual expression for the generating function of a free field with a mass that is the root mean square of m and M. The integration is standard[6] and gives

$$W(J)=\exp\left[-\tfrac{1}{2}i\int d^4x\,d^4x'\,J(x)\,\Delta_F(x,x')J(x')\right], \tag{2.12}$$

where

$$\Delta_F(x,x')=\int\frac{d^4q}{(2\pi)^4}e^{-iq\cdot(x'-x)}\frac{1}{q^2-\frac{1}{2}(m^2+M^2)+i\epsilon}. \tag{2.13}$$

Our procedure has led to a free-field theory with no remnants of the semiclassical bags discussed in Ref. 1. Note that if we try to take M to infinity as done in Ref. 1, then all states but the vacuum acquire infinite energy. This is why we have kept M finite.

Higher terms in the expansion of Eq. (2.9) contain pieces with the form of self-couplings of the ϕ field. However, they are always multiplied by enough powers of the cutoff a that they will vanish as $a\to0$, even taking account of the standard divergences of quantum field theory. For example, one might expect a logarithmic divergence associated with the ϕ^4 term in Eq. (2.9); however, it is multiplied by the cutoff to the fourth power and thus is still removed in the continuum limit of vanishing cutoff. Here we have assumed the same cutoff for the field theory as for the bag integral; to choose a smaller one for the fields would correspond to doing the field integration first, as done in the next section.

Treating the region V classically and the fields ϕ quantum mechanically, the authors of Ref. 1 argue that B needs an infinite renormalization arising from the differing zero-point energies of the field inside and outside the bag. In our approach we have obtained a convergent theory; so no renormalizations are demanded of us. Indeed, the parameter B has disappeared from the theory. However, if we allow the bare B to be infinite as in Ref. 1, we should be more careful with the expansion in Eq. (2.9). To study this question we use the lattice spacing to cut off the divergence in the zero-point energy and calculate the divergent piece of B as suggested in Ref. 1.

Associated with a free field of mass m is a zero-point energy density (π/a is the momentum-space ultraviolet cutoff)

$$\mathcal{H}_0=\frac{1}{4\pi^2}\int_0^{\pi/a} k^2dk\,(k^2+m^2)^{1/2}\,. \tag{2.14}$$

The authors of Ref. 1 argue that the bare bag constant B contains a divergent piece to cancel the difference in this zero-point energy inside and outside the bag. This suggests

$$B=B_R+\frac{1}{4\pi^2}\int_0^{\pi/a} k^2dk[(k^2+M^2)^{1/2}-(k^2+m^2)^{1/2}]$$
$$\underset{a\to 0}{\sim}\frac{M^2-m^2}{16a^2}\,, \tag{2.15}$$

where B_R is a finite quantity. We insert this into the expansion of Eq. (2.9) with the result

$$\sum_i \ln\left[2\cos\left(\int_i d^4x[\tfrac{1}{4}(M^2-m^2)\phi^2-\tfrac{1}{2}B]\right)\right]$$
$$=\Omega\left(\frac{\ln 2}{a^4}-\frac{(M^2-m^2)^2}{2^{11}}\right)+O(a^2)\,. \tag{2.16}$$

The additional constant piece is absorbed in N and we arrive at the previous result in Eq. (2.10).

If we allow a stronger divergence in B one can force additional ϕ-dependent terms to survive as $a\to 0$ in Eq. (2.9). Such terms, however, become effective non-Hermitian couplings in the Lagrangian unless a complex B is taken. We feel that taking a complex B of order the fourth power of the cutoff is outside the spirit in which the model was proposed.

III. ORDERS OF INTEGRATION

In the preceding section we first summed over bag volumes and then integrated over fields. The credibility of our result would be in doubt if this order of doing things was essential. Thus in this section we sum over fields first and then show that the same result follows. In essence we first quantize the field ϕ in an arbitrary classical bag configuration and then perform the path-integral summation over these configurations.

For a given volume V the Lagrangian is a quadratic form in the fields and the ϕ integration is standard,

$$W(J)=N\sum_V\int d\phi\exp\left[i\int d^4x\left(\tfrac{1}{2}(\partial_\mu\phi)^2-\frac{1}{2}\left(\frac{M^2+m^2}{2}\right)\phi^2-\tfrac{1}{2}B+S(x)[\tfrac{1}{4}(M^2-m^2)\phi-\tfrac{1}{2}B]\right)\right]$$
$$=N'\sum_V(\det\Delta_F)^{1/2}\exp\left[-\tfrac{1}{2}i\int d^4x\,d^4x'\,J(x)J(x')\Delta_F(x,x',S)\right]\exp\left(-i\int d^4x\,\tfrac{1}{2}[1+S(x)]B\right)\,, \tag{3.1}$$

where N' is a new normalization factor and $S(x)$ is the function defined in Eq. (2.6). The function $\Delta_F(x,x',S)$ is the propagator for the ϕ field in the presence of the bag configuration V. It satisfies the differential equation

$$\left[\Box_x+\frac{M^2+m^2}{2}-S(x)\left(\frac{M^2-m^2}{2}\right)\right]\Delta_F(x,x',S)=-\delta^4(x'-x) \tag{3.2}$$

and the boundary condition that it vanishes as $|(x-x')_0|\to\infty$ when m^2 is given a small negative imaginary part. We now divide space-time into hypercubes of volume a^4 as in the preceding section. To do the sum over volumes we use the identity

$$f(1)+f(-1)=\left\{\exp\left[\ln\left(2\cosh\frac{d}{dv}\right)\right]f(v)\right\}\bigg|_{v=0} \tag{3.3}$$

to obtain

$$W(J)=N'\exp\left[\sum_i\ln\left(2\cosh\frac{d}{dv_i}\right)\right](\det\Delta_F)^{1/2}$$
$$\times\exp\left\{-\tfrac{1}{2}i\int d^4x\,d^4x'\,J(x)J(x')\Delta_F(x,x',v)-iB\int d^4x\,\tfrac{1}{2}[1+v(x)]\right\}\bigg|_{v=0}, \tag{3.4}$$

where the function $v(x)$

$$v(x)=v_i \tag{3.5}$$

when x is the ith basic hypercube. The propagator $\Delta_F(x,x',v)$ satisfies the equation

$$\left(\Box_x+\frac{m^2+M^2}{2}-v(x)\frac{M^2-m^2}{2}\right)\Delta_F(x,x',v)=-\delta^4(x'-x) \tag{3.6}$$

with the boundary condition $\Delta_F(x,x',v)\to 0$ as $(x-x')_0\to\pm\infty$ and $m^2\to m^2-i\epsilon$.

As a first step in removing the lattice spacing we convert to a continuum notation with the substitution

$$d/dv_i \to a^4\delta/\delta v(x)\,, \tag{3.7}$$

$$\left[\frac{d}{dv_i}, v_j\right] = \delta_{ij} \to \left[\frac{\delta}{\delta v(x)}, v(y)\right] = \delta^4(x-y)\,, \tag{3.8}$$

$$\sum_i \to a^{-4}\int d^4x\,. \tag{3.9}$$

This gives

$$W(J) = N' \exp\left[\int d^4x\, a^{-4}\ln\left(2\cosh a^4 \frac{\delta}{\delta v(x)}\right)\right](\det\Delta_F)^{1/2}\exp\left[-\tfrac{1}{2}i\int dx\,dx'\,J(x)J(x')\Delta_F(x,x',v) - iB\int d^4x\left(\frac{1+v(x)}{2}\right)\right]. \tag{3.10}$$

Expanding the first exponent in powers of a, we find

$$a^{-4}\ln\left(2\cosh a^4\frac{\delta}{\delta v(x)}\right) = a^{-4}\ln 2 + \tfrac{1}{2}a^4\left(\frac{\delta}{\delta v(x)}\right)^2 + O(a^8)\,. \tag{3.11}$$

The first term, $a^{-4}\ln 2$, gives the same infinite constant factor seen in Eq. (2.9) and is absorbed in N'. If we can take a^4 to zero in the second term we obtain

$$W(J) = \exp\left[-\tfrac{1}{2}i\int dx\,dx'\,J(x)J(x')\,\Delta_F(x,x',v)\right]\bigg|_{v=0}$$
$$= \exp\left[-\tfrac{1}{2}i\int dx\,dx'\,J(x)J(x')\,\Delta_F(x,x')\right], \tag{3.12}$$

where $\Delta_F(x,x')$ is the free propagator for the average mass squared as given in Eq. (2.13).

This argument could break down if the $[\delta/\delta v(x)]^2$ in the second term of Eq. (3.11) is sufficiently singular to cancel the a^4 factor. To investigate this we must study the effect of functional differentiation of $\Delta(x,x',v)$ with respect to v. Taking $\delta/\delta v(y)$ of Eq. (3.6) we find

$$\left(\Box_x + \frac{m^2+M^2}{2} - v(x)\frac{M^2-m^2}{2}\right)\frac{\delta}{\delta v(y)}\Delta_F(x,x',v) = \frac{M^2-m^2}{2}\,\delta^4(x-y)\,\Delta_F(x,x',v)\,. \tag{3.13}$$

This is easily solved using (3.6) again

$$\frac{\delta}{\delta v(y)}\Delta_F(x,x',v) = -\frac{M^2-m^2}{2}\,\Delta_F(x,y,v)\,\Delta_F(y,x',v)\,. \tag{3.14}$$

We now work out

$$\frac{\delta}{\delta v(y_1)}\,\frac{\delta}{\delta v(y_2)}\Delta_F(x,x',v) = +\left(\frac{M^2-m^2}{2}\right)^2[\Delta_F(x,y_1,v)\,\Delta_F(y_1,y_2,v)\,\Delta_F(y_2,x',v) + \Delta_F(x,y_2,v)\,\Delta(y_2,y_1,v)\,\Delta_F(y_1,x'v)]\,. \tag{3.15}$$

If we now try to set $y_1 = y_2 = y$, we encounter a short-distance singularity in the propagator $\Delta_F(y_1,y_2,v)$,

$$\frac{\delta}{\delta v(y_1)}\,\frac{\delta}{\delta v(y_2)}\Delta_F(x,x',v) \underset{y_1-y_2\to 0}{\sim} \Delta_F(y_1,y_2,v) \sim \frac{1}{(y_1-y_2)^2}\,, \tag{3.16}$$

so the $[\delta/\delta v(x)]^2$ occurring in (3.11) is indeed singular. However, in this problem we have the cutoff a and these two derivatives actually occur with a point separation of order a. Thus we must consider $[\delta/\delta v(x)]^2$ to be of order a^{-2}. Clearly the a^4 factor in Eq. (3.11) still suffices to remove this term as $a\to 0$. Similar arguments remove higher terms in the expansion of Eq. (3.11). Thus we still obtain Eq. (3.12) confirming the conclusion of the preceding section.

IV. THE BAG WITH SURFACE TENSION

In this section we add to the bag Lagrangian a term giving an effective surface energy to the classical bag. This makes the surface a true dynamical variable. We will show that this is insufficient to adequately control the quantum fluctuations and the theory remains free with the field having an average mass squared.

For a bag theory with surface tension we take the Lagrangian

$$L=\int_V d^3x[\tfrac{1}{2}(\partial_\mu\phi)^2-\tfrac{1}{2}m^2\phi^2-B]+\int_{\bar V} d^3x[\tfrac{1}{2}(\partial_\mu\phi)^2-\tfrac{1}{2}M^2\phi^2]+\int d^3x\,Kn_\mu\partial_\mu S(x)\,. \tag{4.1}$$

Here $S(x)$ is the function defined in Eq. (2.6), n_μ is the inward normal to the bag surface, and K is a constant representing the energy per unit area contained in a static bag boundary. Note that $\partial_\mu S(x)$ is proportional to n_μ times a δ function on the boundary. The surface tension is formally a quadratic form in the derivatives of $S(x)$.

As before we introduce a hypercubical lattice and restrict $S(x)$ to the same value over each fundamental hypercube. At this point the procedure is somewhat ambiguous because of the inherent Lorentz noninvariance of the lattice. Such a noninvariance is routine to lattice formulations of field theories and should disappear in the continuum limit. We proceed by considering the pairs of adjacent hypercubes which have different values of S, i.e., a bag boundary lies between them. For such pairs we introduce in the action a term $-Ka^2$ for spacelike separation or $+Ka^2$ for timelike separation. Thus we are led to consider the action

$$\mathcal{S}=\int d^4x\left[\tfrac{1}{2}(\partial_\mu\phi)^2-\frac{1}{2}\left(\frac{m^2+M^2}{2}\right)\phi^2-\tfrac{1}{2}B\right]+\sum_i S_i\int_i d^4x[\tfrac{1}{4}(M^2-m^2)\phi^2-\tfrac{1}{2}B]-\tfrac{1}{8}Ka^2\sum_{i,j}A_{ij}(S_i-S_j)^2\,, \tag{4.2}$$

where

$$A_{ij}=\begin{cases}+1 \text{ for relatively spacelike adjacent hypercubes}\\ -1 \text{ for relatively timelike adjacent hypercubes}\\ 0 \text{ for nonadjacent hypercubes}\,.\end{cases} \tag{4.3}$$

Note the similarity of the surface-tension term to the Hamiltonian for the Ising model in statistical mechanics.[12] Indeed, the following manipulations should be familiar to devotees of that model. The path integral now reads

$$W(J)=N\sum_{S_i=\pm1}\int d\phi\exp\left[i\left(\mathcal{S}+\int d^4x\,J(x)\phi(x)\right)\right]. \tag{4.4}$$

The sum over S_i is rewritten using Eq. (3.3),

$$W(J)=N\int d\phi\exp\left[\sum_i\ln\left(2\cosh\frac{d}{dv_i}\right)\right]\exp\left\{i\left[\int d^4x\left(\tfrac{1}{2}(\partial_\mu\phi)^2-\frac{1}{2}\left(\frac{m^2+M^2}{2}\right)\phi^2+J\phi-\tfrac{1}{2}B\right)+\sum_i v_i\int_i d^4x[\tfrac{1}{4}(M^2-m^2)\phi^2-\tfrac{1}{2}B]-\tfrac{1}{8}Ka^2\sum_{i,j}A_{ij}(v_i-v_j)^2\right]\right\}\Bigg|_{v_i=0}. \tag{4.5}$$

Now we convert back to a continuum notation using Eqs. (3.7)–(3.9) and the relation

$$-\tfrac{1}{8}Ka^2\sum_{i,j}A_{ij}(v_i-v_j)^2\to+\tfrac{1}{4}K\int d^4x[\partial_\mu v(x)]^2 \tag{4.6}$$

to obtain

$$W(J)=N'\int d\phi\exp\left[a^4\int d^4x\left(\frac{\delta}{\delta v(x)}\right)^2+O(a^8)\right]\times\exp\left\{i\int d^4x\left(\tfrac{1}{2}(\partial_\mu\phi)^2-\frac{1}{2}\left(\frac{m^2+M^2}{2}\right)\phi^2+J\phi\right)+\int d^4x\,v(x)[\tfrac{1}{4}(M^2-m^2)\phi^2-\tfrac{1}{2}B]+\int d^4x\,\tfrac{1}{4}K[\partial_\mu v(x)]^2\right]\Bigg\}\Bigg|_{v=0}, \tag{4.7}$$

where constant factors have been absorbed in N'. If we can now take a to zero in the first exponential, we will recover the result of Secs. II and III, namely the terms involving v all drop out and we have a free theory with mass squared the average of m^2 and M^2. However, just as in the preceding section, we must worry about possible singularities arising from the $[\delta/\delta v(x)]^2$ multiplying a^4. (Higher terms in the expansion involving more powers of a should be less important.) To argue that these cause no trouble, we convert the operations involving v into a path integral over a new local field $\psi(x)$ using the relations (N'' is a divergent normalization)

$$\exp\left[a^4\int d^4x\left(\frac{\delta}{\delta v(x)}\right)^2\right]=N''\int d\psi \exp\left[-\int d^4x\left(a^{-4}[\psi(x)]^2+2\psi(x)\frac{\delta}{\delta v(x)}\right)\right] \tag{4.8}$$

and

$$\exp\left[-\int d^4x\, 2\psi(x)\frac{\delta}{\delta v(x)}\right]f(v)\bigg|_{v=0}=f(-2\psi)\,.$$

Thus $W(J)$ becomes (N''' is a new normalization factor)

$$W(J)=N'''\int d\phi\, d\psi \exp\left[i\int d^4x\left(\tfrac{1}{2}(\partial_\mu\phi)^2-\frac{1}{2}\left(\frac{m^2+M^2}{2}\right)\phi^2+J\phi+K(\partial_\mu\psi)^2-2\psi(x)[\tfrac{1}{4}(M^2-m^2)\phi^2-\tfrac{1}{2}B]+ia^{-4}\psi^2\right)\right]. \tag{4.9}$$

This shows that the theory is equivalent to an interacting field theory with the ψ field having a large imaginary mass squared. Taking a to zero, this mass goes to infinity as a^{-4}. In conventional Feynman perturbation theory this is a strong enough behavior to make all diagrams containing ψ lines go to zero. Consequently, we recover the same free theory for the ϕ field as found in the preceding sections.

V. CONCLUDING REMARKS

In our calculations we only used scalar fields. For Dirac fields the same general results will follow; however, complications arise with vector fields. The boundary conditions of Ref. 1 require that the bag surface be a magnetic source. These conditions do not arise simply by taking the field mass to be large outside the bag; rather, the field is not allowed outside the bag at all. In this paper we have shown for scalar fields that if the external mass goes to infinity, we have a free theory of infinite-mass particles. We conjecture a similar result for non-Abelian gauge fields, namely that the vacuum will be the only finite-energy state for the quantized bag theory with boundary conditions as in Ref. 1.

It may be that some alternative quantization scheme can avoid the difficulties found here. Even if not, our conclusions are dependent on the sharpness of the bag boundary. The bag can still be a useful phenomenological tool if the surface effects are small, which the success of the model supports. If Yang-Mills gauge theory provides the correct explanation for quark confinement through the infrared divergences of the quantum theory, the bag might well provide an approximate classical description of hadronic states.

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