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Roulette Wheels and Quark Confinement

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Monte Carlo simulation of statistical systems represents an ancient tool of the condensed matter physicist. One stores in a computer memory the numerical degrees of freedom of a thermodynamical model. Pseudo-random changes, weighted by Boltzmann's factor, then mimic thermal evolution and fluctuation. The physicist thus performs experiments on "virtual" crystals with a precisely known and freely chosen Hamiltonian. By isolating various features of the dynamics, one hopes to gain insight into such phenomena as phase transitions.

Quite recently, particle theorists have taken this technique over to relativistic quantum field theory and, in particular, to the study of quark confinement in four dimensional gauge theories. Using an analogy to be discussed below, we have been studying four dimensional space-time crystals where the degrees of freedom are gauge fields. These "experiments" have yielded strong numerical evidence that a non-Abelian gauge theory of the nuclear interactions can simultaneously display the phenomena of (1) asymptotic freedom, i.e., weak quark interactions at short distances, and (2) imprisonment of quarks into the physical hadrons.

Confinement by gauge fields in four dimensions can only be a non-perturbative phenomenon. If quarks experience the celebrated linear potential at long distances, then a straightforward renormalization group argument shows an essential singularity in the coefficient of this linear potential when regarded as a function of the gauge coupling constant. Thus quark imprisonment cannot be studied in the conventional Feynman expansion.

The best current evidence for confinement comes from Wilson's formulation of the gauge theory on a lattice.[1] The primary virtue of the lattice is to provide a non-perturbative, short wavelength cutoff at the lattice spacing. Unlike more conventional procedures for removing divergences, the lattice prescription is applied at the outset, and the theory is well defined before any approximation scheme is attempted. However, it should be remembered that this is just a mathematical trick, and a continuum limit must be taken at the end of any calculation of a physical number.

Once formulated on a lattice, the gauge theory becomes a statistical mechanics problem. In this analogy, temperature corresponds to the square of the field theoretical coupling constant. In his original paper, Wilson studied a high temperature expansion and found that confinement arose naturally. This expansion was in terms of quarks at the ends of flux strings which carry a finite energy per unit length. This gives directly an interquark potential which grows linearly with separation.

For particle physics we are not directly interested in the strong coupling regime, but rather in the continuum limit. From renormalization group analysis one derives the phenomenon of asymptotic freedom, whereby the effective coupling constant of a non-Abelian gauge theory decreases logarithmically as the scale of measurement goes to zero.[2] The coupling constant appearing in the Wilson expansion represents the bare coupling at the scale of the lattice spacing. Asymptotic freedom thus indicates that this coupling should go to zero for a continuum limit, forcing us to leave the strong coupling domain.

In the statistical mechanical analog, we know there is confinement at high "temperature", but we need low temperature properties. Many statistical systems undergo phase transitions as the temperature is varied, and distinct phases may have qualitatively different bulk properties (i.e., ice vs. water). A demonstration of confinement would be greatly simplified if four dimensional non-Abelian gauge theories have no phase transitions separating strong from weak coupling. For electrodynamics a different situation ensues. Here Wilson's expansion also gives confinement at strong coupling, but we know that photons are massless and electrons unconfined. The entire validity of the Wilson approach hinges on the appearance of a deconfining phase transition in U(1) lattice gauge theory. Such a transition has been seen in Monte Carlo simulations and its existence proved rigorously.[3,4]

What has enabled us to use statistical mechanical methods to study gauge theories is the Feynman path integral formulation of quantum mechanics. Indeed, a Feynman path integral is formally nothing but a partition function for an equivalent statistical system. In our Monte Carlo approach we are attempting to numerically evaluate such integrals using established statistical methods.

A path integral is highly multidimensional. For example, on a 10^4 site lattice (typical of later discussion) the integral for SU(2) lattice gauge theory with three four-vector potentials on each site has 1.2×10^5 dimensions. This precludes conventional integration routines but strongly suggests a statistical treatment. A Monte Carlo procedure generates a sequence of field configurations by random changes in the elements of this sequence. The algorithm is constructed in such a manner that ultimately the probability of any particular configuration appearing in this sequence is proportional to the Boltzmann weighting. In the field theory this weighting is the exponential of the action for that configuration. Thus we are essentially bringing our lattices into "thermal equilibrium" with a heat bath at a temperature corresponding to the bare coupling constant.

The procedure, then, reduces to doing "experiments" with "crystals" stored in a computer memory. These crystals are four dimensional because of the unification of space and time in a relativistic theory. One advantage of this method is that the entire lattice is stored. Thus one can reconstruct and study any desired correlation function. Thusfar our systems have been rather small, up to 10^4 sites for SU(2) and 6^4 sites for SU(3). Recent investigations of a discrete approximation to SU(2) have used a 16^4 lattice.[5]

I will now display the results of some simple experiments.[3,6] Figure 1 exhibits several runs with the gauge group SU(2) at a particular coupling $\beta = \frac{4}{g_0^2} = 2.3$. This value was selected because it represents essentially the worst convergence for SU(2) formulated with Wilson's action. Runs are shown for 4^4 to 10^4 site lattices. I have plotted the "average plaquette" which is just the internal energy of the system or the expectation value of the gauge field strength squared. This is plotted agains the number of Monte Carlo sweeps performed on the lattice. For each size lattice, two different initial configurations were taken, one totally ordered with vanishing vector potential and the other with totally random values for the fields. Thus we are approaching equilibrium from opposite extremes, zero and infinite temperature. Agreement of these two runs is a test of equilibrium. Note that for all lattices convergence is essentially complete after only 20-30 iterations. Thermal fluctuation are apparent on the smallest systems, but they are relatively small on the 10^4 site crystal.

Figure 2 shows the evolution of the internal energy from an ordered start and at several values of coupling. Note that convergence gets extremely good both at strong and weak coupling. Thus this technique is tied to neither regime and interpolates nicely between both.

The situation can be much worse if a phase transition is nearby. Figure 3 shows a Monte Carlo run from an ordered start for a Z_2 gauge theory.[7] This is a toy model where each component of the gauge potential can take on only two values. This system is a useful theoretical laboratory because duality arguments indicate a phase transition at a known value of coupling.[8] In this figure we are heating the system through this phase transition. Note that this takes several hundred iterations and on the way there is a tendency to be hung up in a metastable phase. Figure 4 shows runs with ordered and disordered starts exactly at the critical temperature. Note that the points do not converge and the system appears to have two distinct stable phases. This is evidence for a first order phase transition in this model.

In Fig. 5 I show rapid thermal cycles on SU(2) gauge theory in 4 and 5 space-time dimensions and U(1) gauge theory in 4 dimensions. Each point was obtained by heating or cooling the system for on the order of twenty iterations. Phase transitions are to be suspected in regions where the heating and cooling points do not agree. Such "hysteresis" phenomena are apparent for the 5 dimensional SU(2) and the four dimensional U(1) models. More detailed analysis by Lautrup and Nauenberg[9] has indicated that the U(1) transition is second order. Analysis of my own has indicated a first order transition in the 5 dimensional SU(2) case.[3] This clear transition shows the critical nature of 4 dimensions where no strong structure is seen for SU(2).

Once the lattices are in equilibrium, anything of interest can, in principle, be measured. Being interested in a possible linear potential between widely separated quarks

$$E \to Kr \ , \qquad (1)$$

I have attempted to extract the coefficient K by inserting sources with quark quantum numbers into the crystals. Measuring distances in units of the lattice spacing a, I measure the dimensionless combination a^2K. In Fig. 6 I plot[6] the measured values for a^2K as a function of $\beta \equiv 4/g_0^2$ for the gauge group SU(2). On this graph are two theoretical curves. The first term in Wilson's strong coupling expansion gives a K approaching $-\ln(\beta/4)$ as β goes to zero. This is nicely confirmed by the numerical results. The other curve, an exponential falloff of a^2K for large β, is a prediction of asymptotic freedom. Indeed, this is the essential singularity in g_0^2 which prevents a perturbative study of confinement. If a^2K falls faster than the prediction, the interquark potential is weaker than linear at long distances, whereas if it falls more slowly, the confining potential is stronger. The consistency of the numerical results with this theoretical prediction is evidence that a linear potential does indeed survive a continuum limit.

One physical number arises from this analysis. Although asymptotic freedom predicts the exponential decrease of a^2K as β grows, the normalization is undetermined. This normalization relates the strength of the linear long-distance potential to the scale of the logarithmic decrease of the effective gauge coupling at short distances. Using the estimate $\sqrt{K} = 400$ MeV from Regge phenomenology, I have found[10] for SU(3)

$$\Lambda_{MOM} = 170 \pm 50 \text{ MeV} \tag{2}$$

Here Λ_{MOM} is the renormalization scale defined in terms of the three gluon vertex in Feynman gauge. Phenomenological analysis of deep inelastic scattering data can in principle determine Λ_{MOM}, and the currently most popular value is about 800 MeV. The discrepancy between Eq. (2) and this number may be due to the neglect of light virtual quark loops in the Monte Carlo analysis, or it may come from uncertainties in the experimental analysis. In any case, Eq. (2) represents a non-perturbative calculation of a number relating long and short range properties of an interacting field theory.

In Fig. 7 I return to the gauge group SU(2) and plot two functions, F and G, as functions of the bare coupling g_0 squared.[11] Here the function F is proportional to the square of an effective coupling at twice the lattice spacing and G corresponds to four lattice spacings. Note that away from $g_0^2 = 0$ the effective coupling is always larger at the larger scale and consequently there is no evidence of any non-trivial renormalization group fixed point, where F and G would cross. This figure is the strongest evidence that the SU(2) theory does not possess any conventional second order phase transition. When the physical scale is doubled at weak coupling, asymptotic freedom tells how the renormalized charge must change. In Fig. 8 I show these functions F and G again, but with G shifted by the predicted amount. The fact that F and G now fall on top of each other is a numerical verification of asymptotic freedom on these rather small lattices.

I have described only a few highlights of recent research. Other investigations include searches for useful discrete approximation to continuous non-Abelian groups,[5,12] studies of gauge theories at finite physical temperatures,[13] analyses in dimensions other than four,[14] and use of these numerical techniques on ordinary one degree of freedom quantum mechanics.[15] Perhaps the most interesting and frustrating remaining problem is the inclusion of quarks in the calculation. Inclusion of scalar fields is no problem[16] but the generalization to fermi fields is not at all obvious. This is because the action becomes an operator in Grassman space and the path integral becomes a sum over fermion loops. Recent courageous attempts at evaluating these integrals are, as yet, too demanding for computer time to be practical on any but the most modest lattices.[17] I look forward to a technical breakthrough in this area.

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Figure Captions

Fig. 1. The internal energy for SU(2) gauge theory at $\beta = 2.3$ as a function of number of Monte Carlo iteration.

Fig. 2. The evolution of the average plaquette at several values of β.

Fig. 3. Evolution of the Z_2 gauge theory from an ordered start at $\beta = 0.425$. This system has a phase transition at $\beta = 1/2 \ln(1 + \sqrt{2}) = 0.44 \ldots$.

Fig. 4. The evolution of ordered and disordered states at $\beta = \beta_c$ for Z_2 gauge theory.

Fig. 5. The average plaquette as a function of β in a thermal cycle on a) SU(2) in five dimensions; b) SU(2) in four dimensions and c) $SO(2) \equiv U(1)$ in four dimensions. Crosses, heating; circles, cooling.

Fig. 6. The combination a^2K for SU(2) gauge theory as a function of β.

Fig. 7. Effective couplings at twice and four times the lattice spacing as a function of the inverse of the bare charge squared.

Fig. 8. Testing asymptotic freedom.

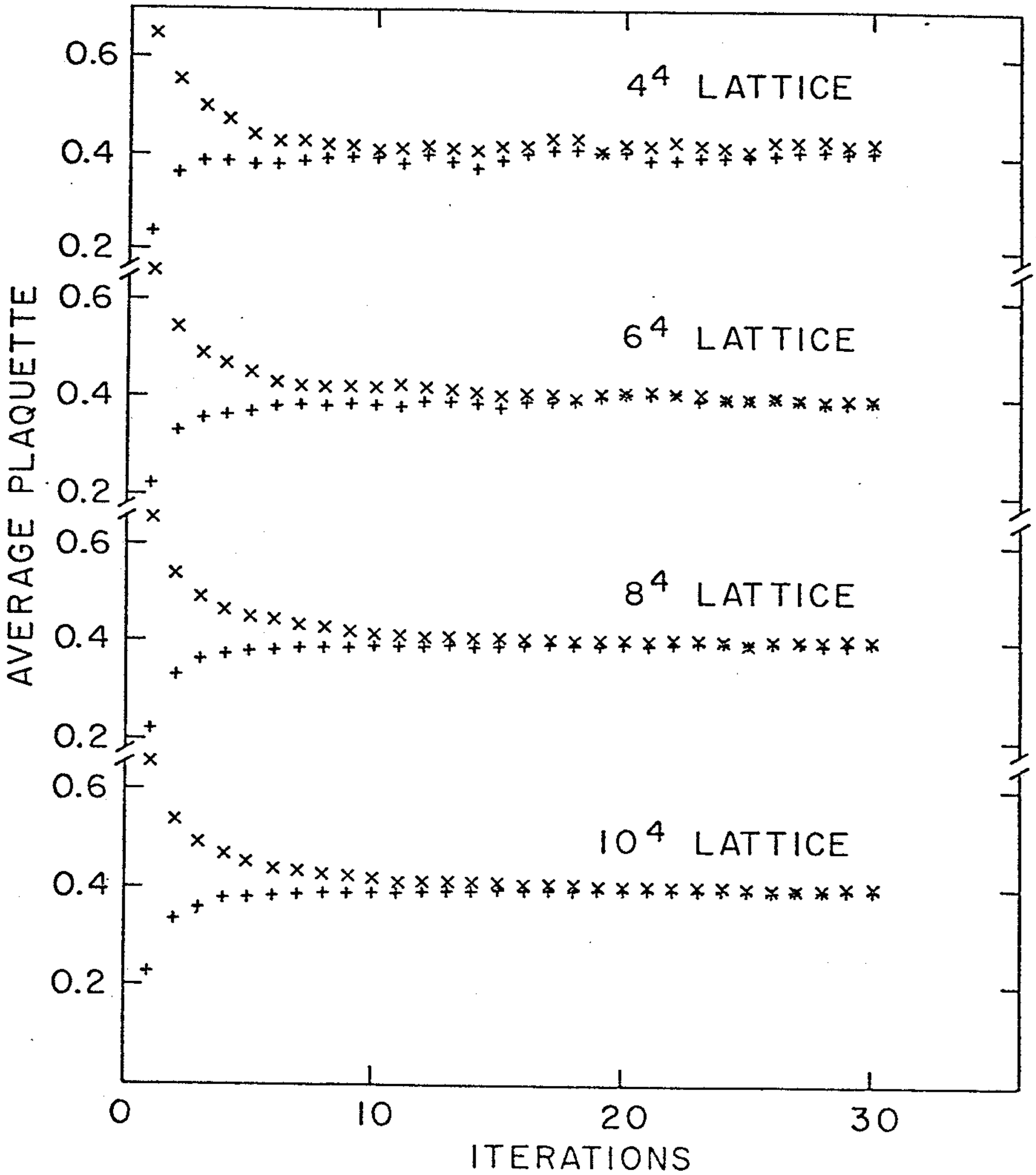
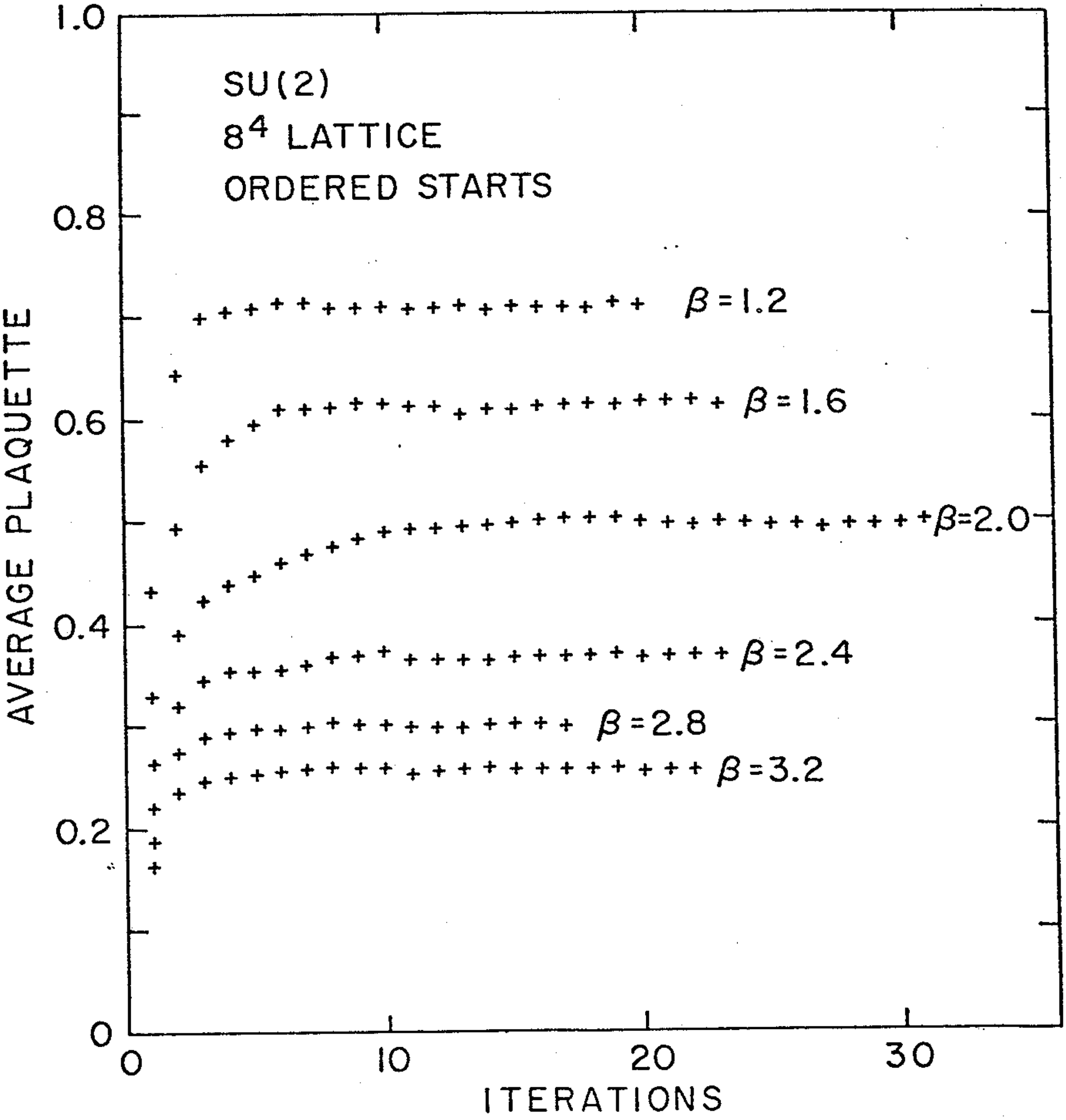

AVERAGE PLAQUETTE
0.6
0.4
0.2
4^4 LATTICE
6^4 LATTICE
8^4 LATTICE
10^4 LATTICE
0
10
20
30
ITERATIONS

Figure 1



SU(2)
8⁴ LATTICE
ORDERED STARTS
β=1.2
β=1.6
β=2.0
β=2.4
β=2.8
β=3.2
AVERAGE PLAQUETTE
ITERATIONS
1.0
0.8
0.6
0.4
0.2
0
0
10
20
30

Figure 2.

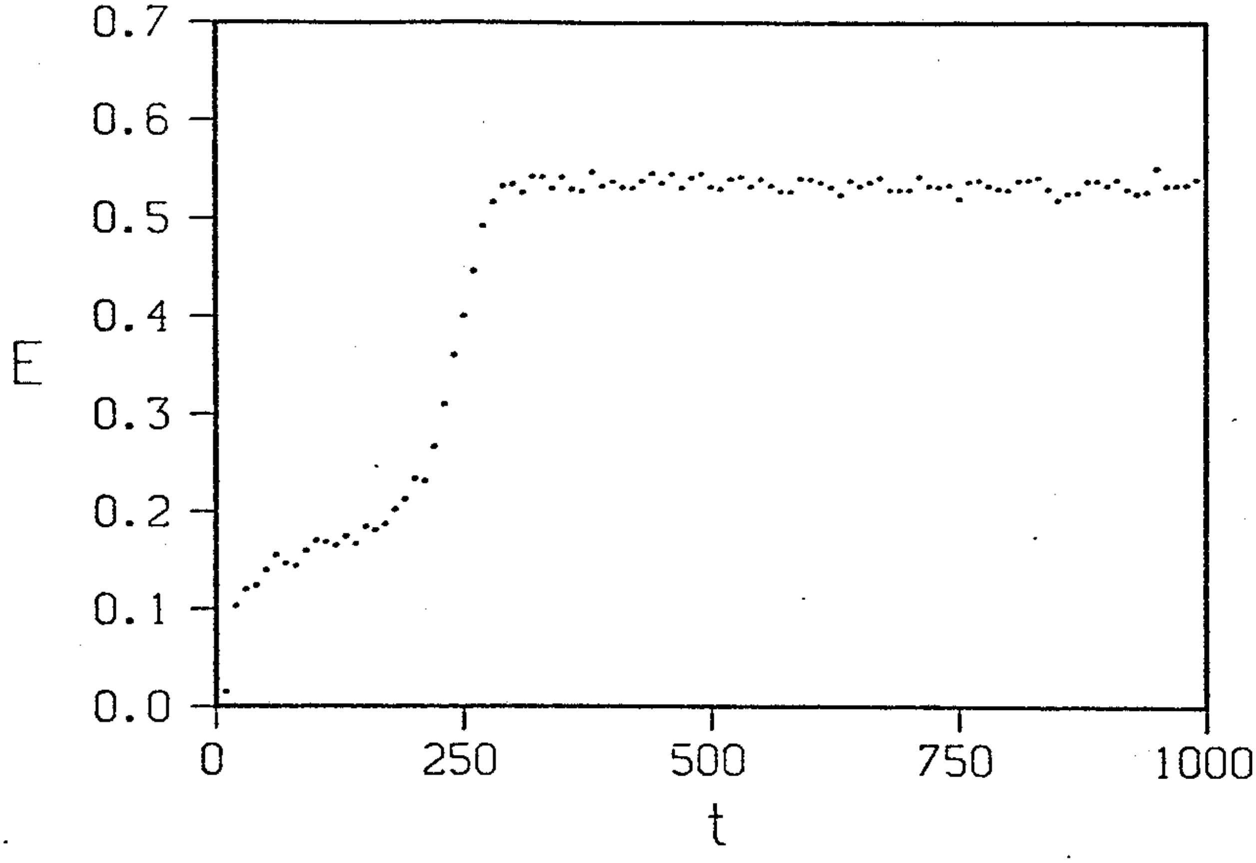


Figure 3

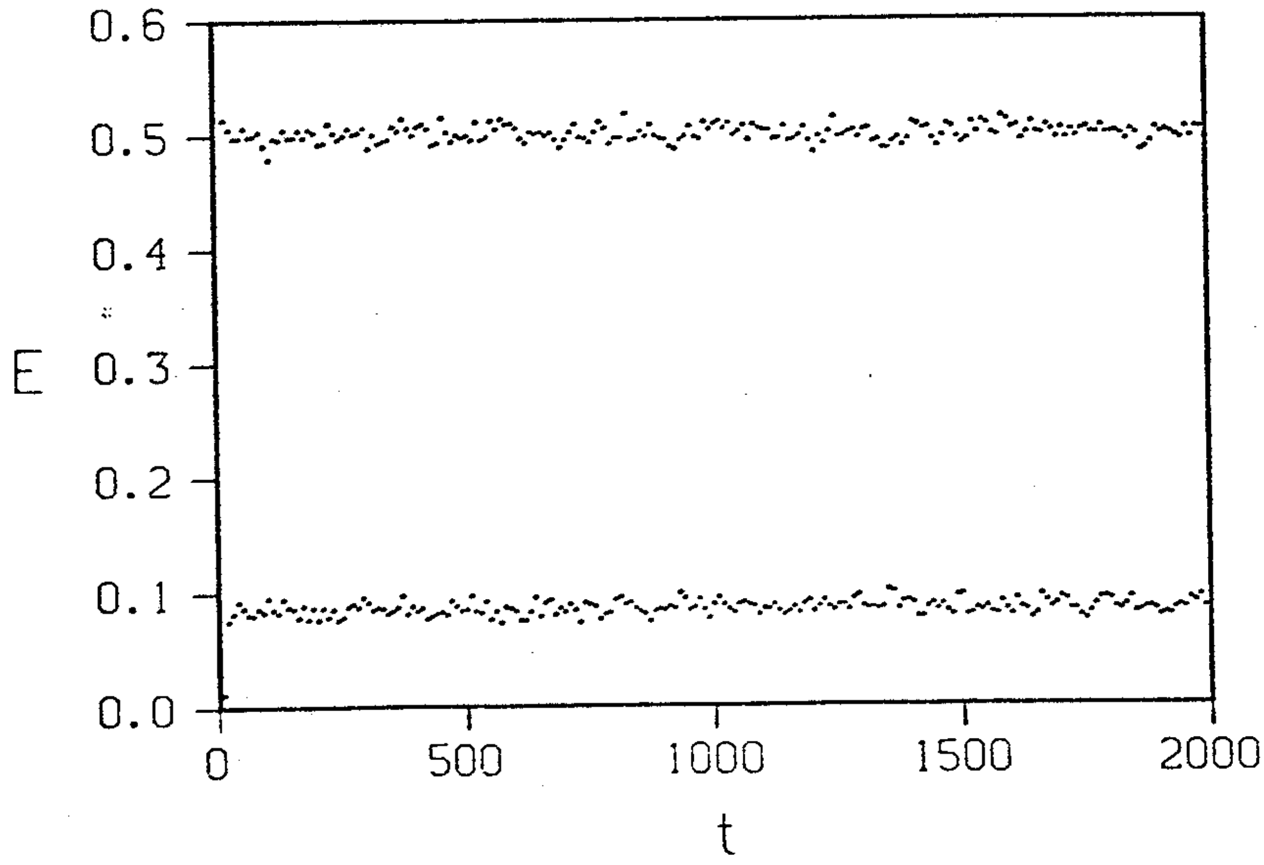


Figure 4

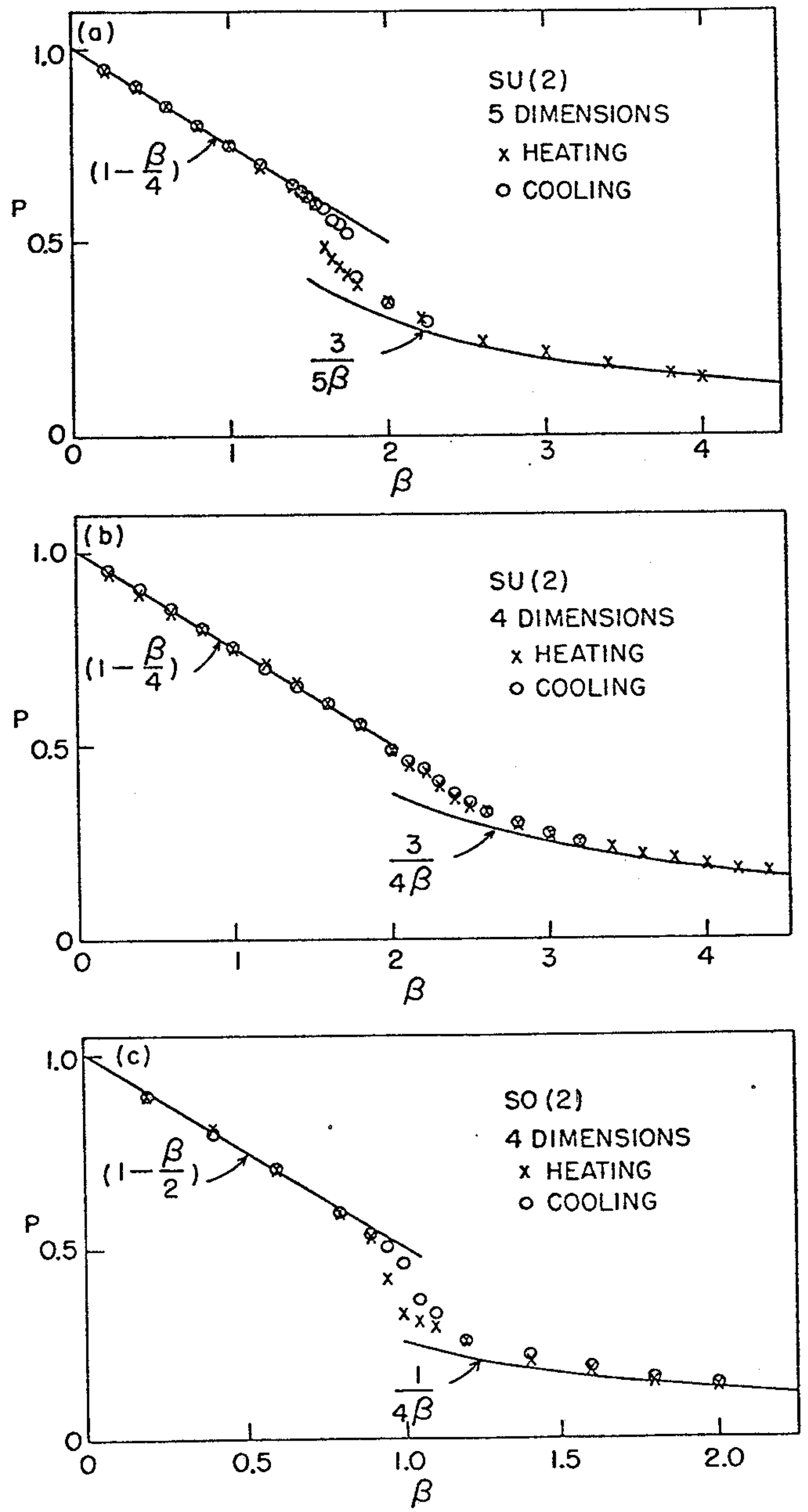

(a)
SU(2)
5 DIMENSIONS
x HEATING
o COOLING
$(1-\frac{\beta}{4})$
$\frac{3}{5\beta}$
P
β
(b)
SU(2)
4 DIMENSIONS
x HEATING
o COOLING
$(1-\frac{\beta}{4})$
$\frac{3}{4\beta}$
P
β
(c)
SO(2)
4 DIMENSIONS
x HEATING
o COOLING
$(1-\frac{\beta}{2})$
$\frac{1}{4\beta}$
P
β

Figure 5

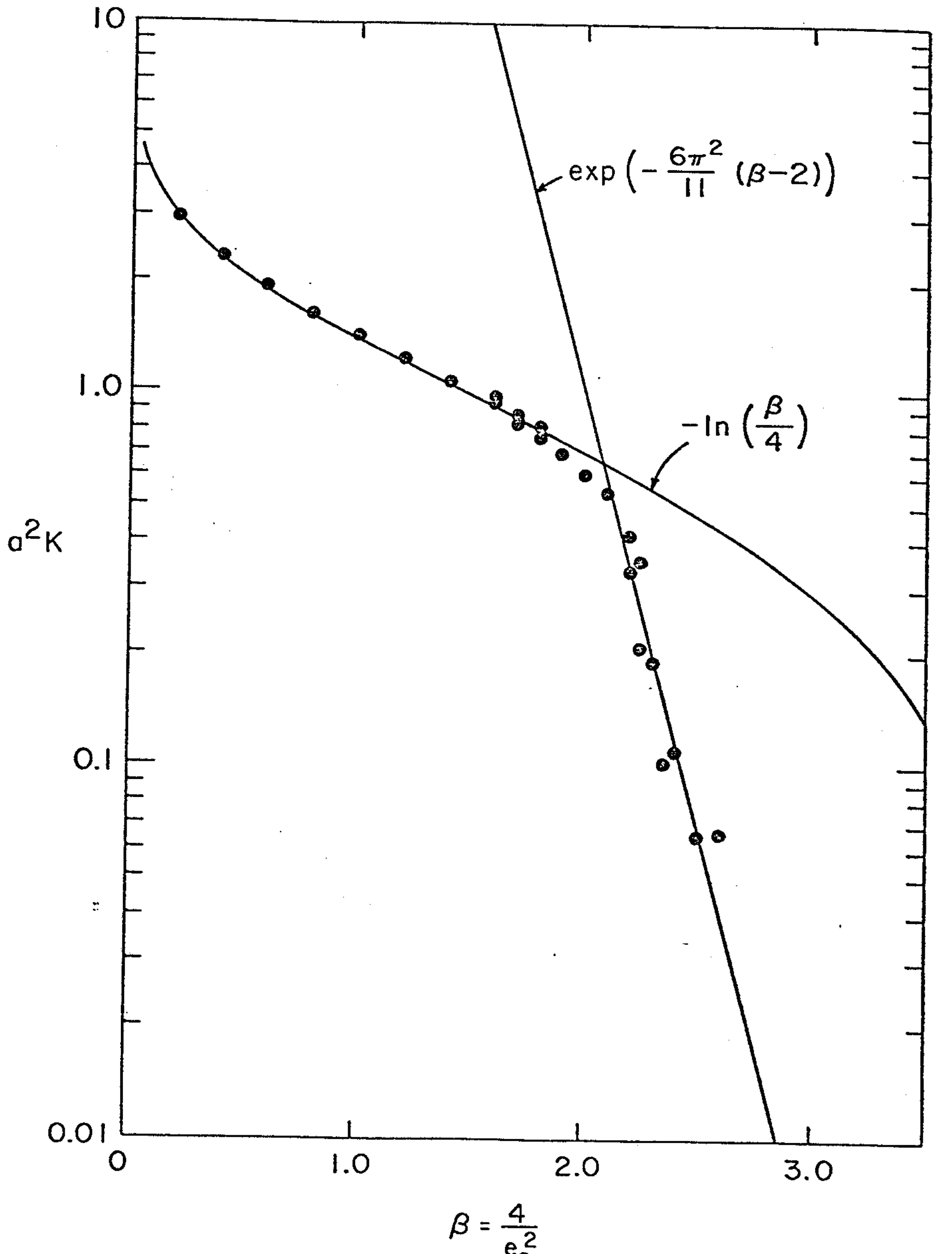

10
1.0
0.1
0.01
a^2K
$\exp\left(-\frac{6\pi^2}{11}(\beta-2)\right)$
$-\ln\left(\frac{\beta}{4}\right)$
0
1.0
2.0
3.0
$\beta = \frac{4}{e_0^2}$

Figure 6

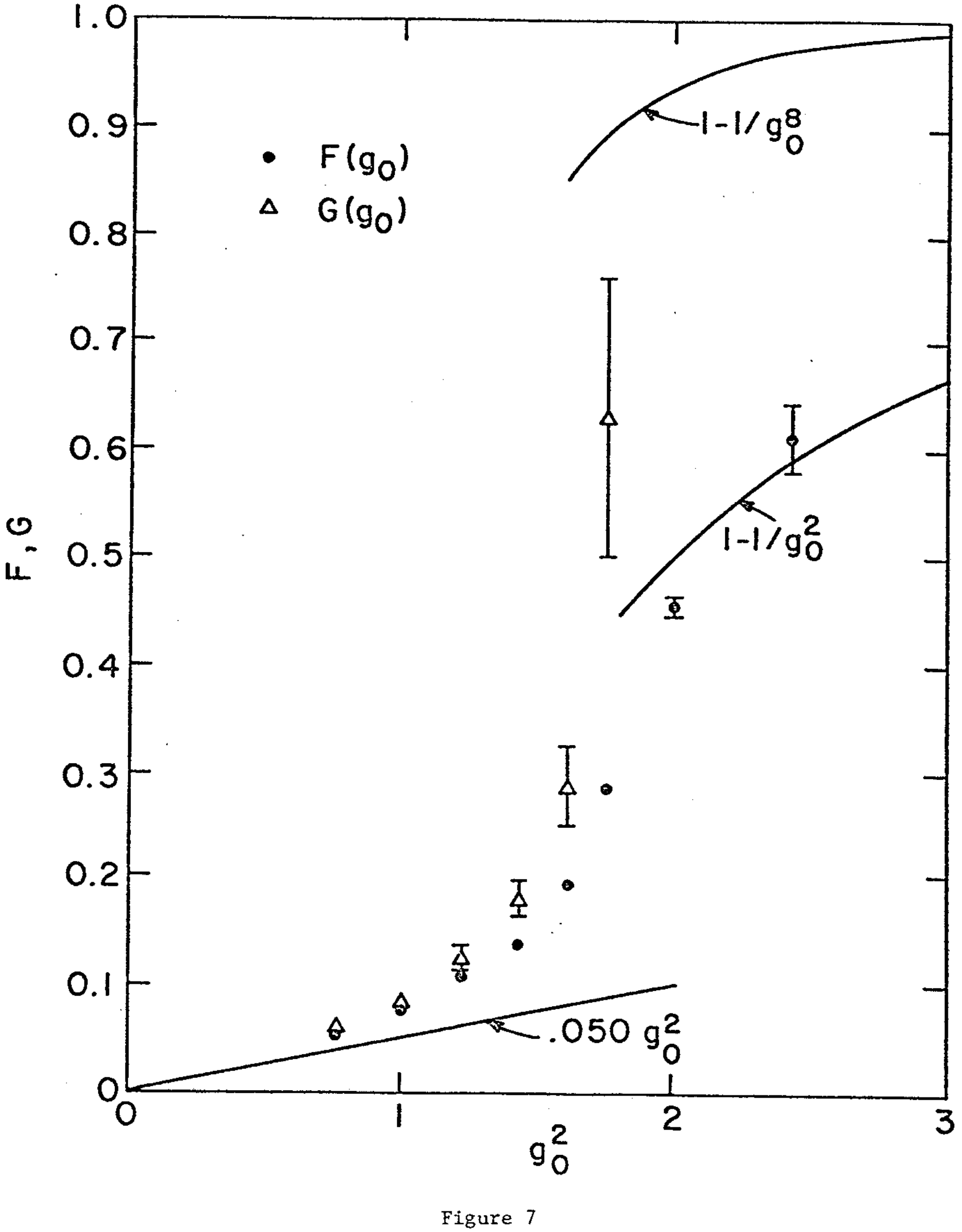

1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
F, G
F(g0)
G(g0)
1-1/g0^8
1-1/g0^2
.050 g0^2
0
1
2
3
g0^2

Figure 7

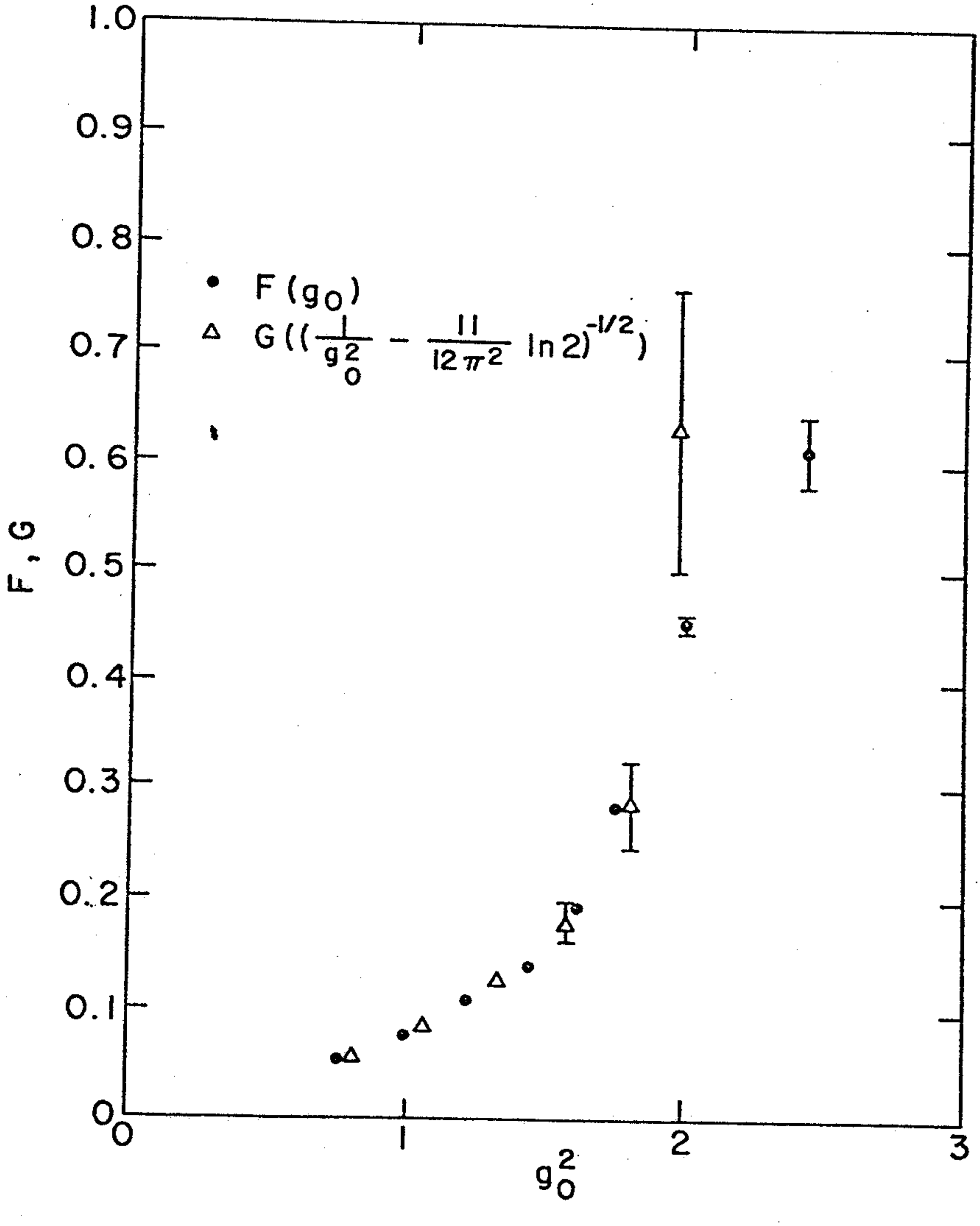


Figure 8