Brief Reports

Brief Reports are short papers which report on completed research which, while meeting the usual **Physical Review** *standards of scientific quality, does not warrant a regular article. (Addenda to papers previously published in the* **Physical Review** *by the same authors are included in Brief Reports.) A Brief Report may be no longer than 3½ printed pages and must be accompanied by an abstract. The same publication schedule as for regular articles is followed, and page proofs are sent to authors.*

Phase transition in SU(6) lattice gauge theory

Michael Creutz and K. J. M. Moriarty*
Brookhaven National Laboratory, Upton, New York 11973
(Received 6 July 1981; revised manuscript received 9 December 1981)

Previous Monte Carlo studies of SU(4) and SU(5) lattice gauge theory are extended to pure SU(6) gauge fields in four space-time dimensions. Using Wilson's form of the action, a first-order phase transition is clearly seen at $\beta_c=12/g_0{}^2=24.0\pm1.0$ on a 4^4 lattice.

It is generally believed that the non-Abelian SU(N) gauge groups in four space-time dimensions confine static quarks for all values of the coupling. It would be simplest if there were no phase transitions in going from the high-temperature region to the low-temperature region. Monte Carlo studies[1] of SU(2) and SU(3) gauge theories have verified this conjecture. However, this behavior has been shown not to hold for SO(3),[2,3] SU(4),[4,5] and SU(5)[4,6] gauge groups in four space-time dimensions where first-order phase transitions were found. Of course, these transitions may be just a transition from one confining phase at high temperature to another confining phase at low temperature and, thus, not lead to deconfinement for any value of the coupling. Here we extend the analysis to SU(6) to see if the first-order phase transition of SU(4) and SU(5) gauge theories survives in SU(6) gauge theory and, if it does, to calculate its critical temperature.

A hypercubical lattice of four Euclidean space-time dimensions with fixed finite lattice spacing is used to define our system. Denoting nearest-neighbor lattice sites by i and j, we then associate with the link $\{i,j\}$ joining them a unitary unimodular $N\times N$ matrix U_{ij}. We define our partition function by

$$Z(\beta)=\int\left[\prod_{\{i,j\}}dU_{ij}\right]\exp(-\beta S[U])\,,$$

where the inverse temperature β is related to the bare coupling constant g_0 by $\beta=2N/g_0{}^2$. The measure in the partition function is the normalized invariant group measure. The action[7] is the sum over all plaquettes $\square$ in the lattice

$$S[U]=\sum_{\square}S_{\square}=\sum_{\square}\left[1-\frac{1}{N}\mathrm{Re\,Tr}U_{\square}\right]\,, \qquad (1)$$

where $U_{\square}$ is the product of link variables around a plaquette. Periodic boundary conditions are used throughout the calculation. Once the system is in equilibrium,[8] we measure the average action per plaquette $\langle E\rangle$. Further details on the calculational procedure can be found in Ref. 4. The high-temperature expansion for the average action per plaquette gives ($N>3$)

$$\langle E\rangle=1-\frac{\beta}{2N^2}+O(\beta^3)\,, \qquad (2)$$

while the low-temperature expansion gives

$$\langle E\rangle=\frac{N^2-1}{4\beta}+O(\beta^{-2})\,. \qquad (3)$$

In Fig. 1(a) we show the average action per plaquette as a function of the number of Monte Carlo iterations for the SU(6) gauge group on a periodic 4^4 hypercubical lattice for both ordered and disordered starting lattices. These runs correspond to an inverse temperature of $\beta=24.0$. From Fig. 1(a) we see that the ordered and disordered starts do not converge to a unique value of the average action per plaquette after 470 iterations but approach two distinct values, suggesting a first-order phase transition in the SU(6) model. By way of contrast, we show the average action per plaquette for both

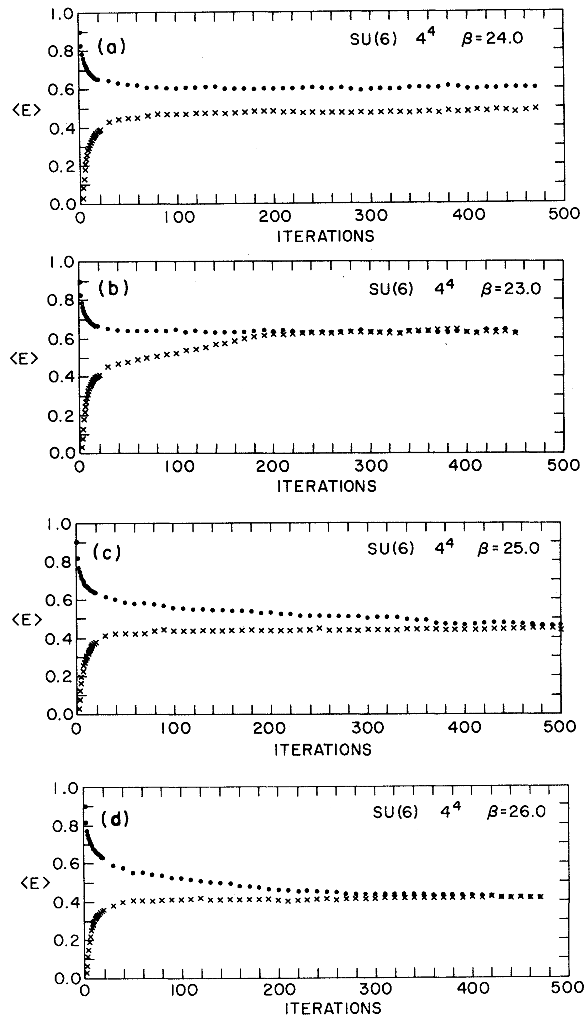

FIG. 1. The evolution of the ordered (crosses) and disordered (full circles) configurations for the SU(6) gauge group on a 4^4 lattice at inverse temperatures of (a) $\beta=24.0$, (b) $\beta=23.0$, (c) $\beta=25.0$, and (d) $\beta=26.0$.

ordered and disordered starting lattices for inverse temperatures of $\beta=23.0$ [Fig. 1(b)], $\beta=25.0$ [Fig. 1(c)], and $\beta=26.0$ [Fig. 1(d)]. We can quite clearly see that the ordered and disordered starts stabilize at unique values of the average action per plaquette after 240, 490, and 420 iterations through a 4^4 lattice for $\beta=23.0$, 25.0, and 26.0, respectively.

In Fig. 2 we show the average action per plaquette for SU(6) as a function of the inverse temperature β. The data points result from 100 iterations through the lattice where we averaged over the last 20 iterations. A hysteresis loop is clearly visible beginning at $\beta \simeq 21.0$. The critical inverse temperature is estimated from the hysteresis loop to be $\beta_c=24.0\pm1.0$. Fig. 1(a) confirms that this is indeed the critical inverse temperature. Also

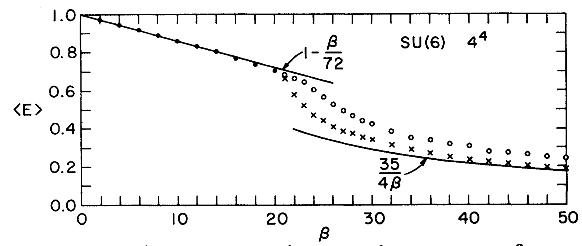

FIG. 2. The average action per plaquette as a function of the inverse temperature β. The crosses and open circles represent the average over the last 20 iterations of 100 iterations through the lattice for ordered and disordered starting lattices, respectively, while the full circles represent cases where the ordered and disordered starting lattices converge to a unique value before 100 iterations through the lattice.

shown in Fig. 2 are the leading-order high- and low-temperature expansions of Eqs. (2) and (3). We can see that our Monte Carlo-generated data is in good agreement with these expansions outside the region of the critical point.

It is well known that the Wilson form of the action is not unique. Many forms of the action will yield the same continuum theory. In particular,[9] in Eq. (1) we can replace Re Tr $U_\square$ by $[\mathrm{Re}[\mathrm{Tr}U_\square^m]^n]^p$, where m, n and, p are integers. Correctly normalized they should all yield the same continuum theory. More generally, we are not limited to looking at an elementary plaquette but can replace our elementary plaquette by any convex polygon of plaquettes and we will again recover the same continuum theory. It may well be that our first-order phase transition found in

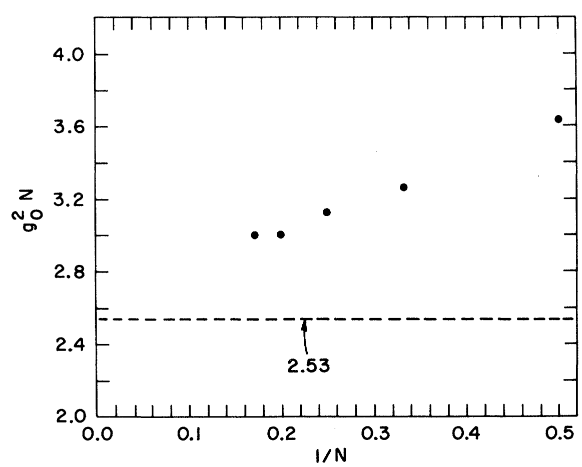

FIG. 3. The critical values of the quantity $g_0{}^2N$ for SU(N) gauge theory as a function of $1/N$.

SU(4), SU(5), and now SU(6) gauge theories are simply a result of taking the Wilson form of the action. Indeed, modification of the SU(2) action can induce a spurious transition, without affecting the continuum physics.[3] We are currently studying similar modifications of the SU(5) action.

It has recently been proposed by Green and Samuel[10] that the SU(N) gauge groups on a four-dimensional space-time lattice would suddenly develop a phase transition at $N=\infty$ with ${g_0}^2N=2.53$. Their conclusion is apparently incorrect in that a first-order transition is already present at finite N. Although the connection with their analysis is unclear, we show in Fig. 3 the critical values of ${g_0}^2N$ as a function of $1/N$ for $N=2$, 3, 4, 5, and 6. In $N=2$ and 3, where no transitions are observed, we plot the position of the peaks in the specific heat.[1] The results are consistent with approaching the value predicted in Ref. 10.

ACKNOWLEDGMENTS

One of the authors (K. J. M. M) wishes to thank the Science Research Council of Great Britain for the award of a Senior Visting Fellowship (Grant NG 10804) to travel to Brookhaven and the Brookhaven Directorate for the award of a Visiting Fellowship to visit Brookhaven where this work was carried out.

*Permanent address: Department of Mathematics, Royal Holloway College, Englefield Green, Surrey, TW20 OEX, United Kingdom.

[1]M. Creutz, Phys. Rev. Lett. 45, 313 (1980); C. Rebbi, Phys. Rev. D 21, 3350 (1980); B. Lautrup and M. Nauenberg, Phys. Rev. Lett. 45, 1755 (1980); K. J. M. Moriarty, Nucl. Phys. (to be published).

[2]I. G. Halliday and A. Schwimmer, Phys. Lett. 101B, 327 (1981); J. Greensite and B. Lautrup, Phys. Rev. Lett. 47, 9 (1981); I. G. Halliday and A. Schwimmer, Phys. Lett. 102B, 337 (1981).

[3]G. Bhanot and M. Creutz, Phys. Rev. D 24, 3212 (1981).

[4]M. Creutz, Phys. Rev. Lett. 46, 1441 (1981).

[5]K. J. M. Moriarty, Phys. Lett. 106B, 130 (1981).

[6]H. Bohr and K. J. M. Moriarty, Phys. Lett. 104B, 217 (1981).

[7]K. G. Wilson, Phys. Rev. D 10, 2445 (1974).

[8]N. Metropolis, A. W. Rosenbluth, M. N. Rosenbluth, A. H. Teller, and E. Teller, J. Chem. Phys. 21, 1087 (1953).

[9]J. G. Taylor, private communication.

[10]F. Green and S. Samuel, Nucl. Phys. B194, 107 (1982).