

MONTE CARLO STUDIES OF QUARK-GLUON DYNAMICS*

Michael Creutz⁺

Brookhaven National Laboratory, Upton, NY 11973

ABSTRACT

Recent progress in understanding strong interaction physics through Monte Carlo simulation of lattice gauge theory is reviewed.

Because of the analogy between the Feynman path integral and a partition function, Monte Carlo simulation of statistical systems has recently become a powerful tool of the elementary particle theorist. This numerical method for the evaluation of path integrals is generally studied with a space time lattice providing an ultraviolet cut-off. The technique converges well in both strong and weak coupling regimes and interpolates nicely in between. Furthermore, as entire field configurations are stored in the computer memory, any desired correlation function is in principle available.

Of course any technique has its limitations. Statistical errors in any Monte Carlo study drop only with the square root of the computer time and extraction of some numbers, such as the glueball mass, has turned out to be highly statistics limited. Also, in four dimensions the linear lattice dimension is necessarily limited; the largest thus far simulated having 16 sites on a side.¹ Finally, although fermionic fields are being studied extensively, the techniques for handling Grassmann variables are as yet rather awkward.

Before proceeding I would like to emphasize one well known but not fully appreciated point about the standard strong interaction theory of quark gluon dynamics. In the chiral limit, when the pseudoscalar meson masses vanish, this theory has no free parameters. All dimensionless quantities, such as the ratio of the ρ mass to the nucleon mass, are in principle determined. This applies even to quantities such as the pion nucleon coupling constant, once considered a possible basis for a perturbative analysis. This beautiful feature of the strong interaction theory is in sharp contrast to the plethora of parameters in theories of the weak and electromagnetic interactions and unification thereof.

The idea of a zero parameter theory is in a way rather frightening. If we calculate an observable and get the number wrong, there is nothing left to adjust. If the delta-nucleon mass splitting comes out wrong, quark gluon dynamics must be abandoned.

Returning now to the lattice theory, the lattice spacing provides a natural scale on which to measure dimensionful quantities. In

⁺Talk at the Division of Particles and Fields of the American Physical Society Meeting, Santa Cruz, September 9-11, 1981

* This work was supported by the U. S. Department of Energy under Contract number DE-AC02-76CH00016.

particular, the inverse of the correlation length ξ in lattice units is the mass m of the lightest state in the theory times the lattice spacing a

$$\xi^{-1} = ma \quad (1)$$

In the continuum limit we wish to take the lattice spacing to zero but have physical masses remain finite. Thus the correlation length goes to infinity and we are driven to a critical point of the analogous statistical system. Using asymptotic freedom, we can trade the lattice spacing with the bare coupling as the parameter of the lattice theory. Indeed, we know that the bare coupling, which is an effective coupling at the scale of the lattice spacing, decreases logarithmically with the cutoff.

$$g_0^2(a) = (\beta_0 \ln(1/\Lambda_0^2 a^2) + (\beta_1/\beta_0) \ln \ln(1/\Lambda_0^2 a^2) + O(g_0^2))^{-1} \quad (2)$$

Here β_0 and β_1 are the first two terms in the Gell-Mann Low function

$$a \frac{d}{da} g_0(a) = \beta_0 g_0^3 + \beta_1 g_0^5 + O(g_0^7) \quad (3)$$

and have been calculated perturbatively.² The dimensionful parameter Λ_0 is an integration constant and provides a natural physical scale in which to measure dimensionful observables. Its value will depend on the details of the cutoff scheme but a perturbative analysis can relate any convention to some standard one.³

We can now take eq. (2) and solve it for the lattice spacing and thus obtain a prediction for the weak coupling behavior of the inverse correlation length

$$ma = \frac{m}{\Lambda_0} (\beta_0 g_0^2)^{-\beta_1/2\beta_0} \exp\left(\frac{-1}{2\beta_0 g_0^2}\right) \times (1 + O(g_0^2)) \quad (4)$$

Note the essential singularity at vanishing coupling. This shows that the spontaneous generation of masses in quark-gluon dynamics is inherently non-perturbative. The basic idea of a lattice calculation is to measure some dimensionful quantity in units of the lattice spacing and to look for the coupling dependence indicated in eq. (4). The physical value of the parameter in units of Λ_0 is then the coefficient of this behavior.

Although we eagerly await results from the full theory, most work has thus far been on the pure gluon sector of the theory. Also, because of the extra complications involved in SU(3), much work has concentrated on the simpler non-Abelian gauge group SU(2). The first dimensional parameter extracted from the pure glue theory was the coefficient of the long range linear interquark potential. In fig. 1 we show an effective force $\chi(I,I)$ between two quarks separated by I lattice spacings.⁴ These curves form an envelop representing the

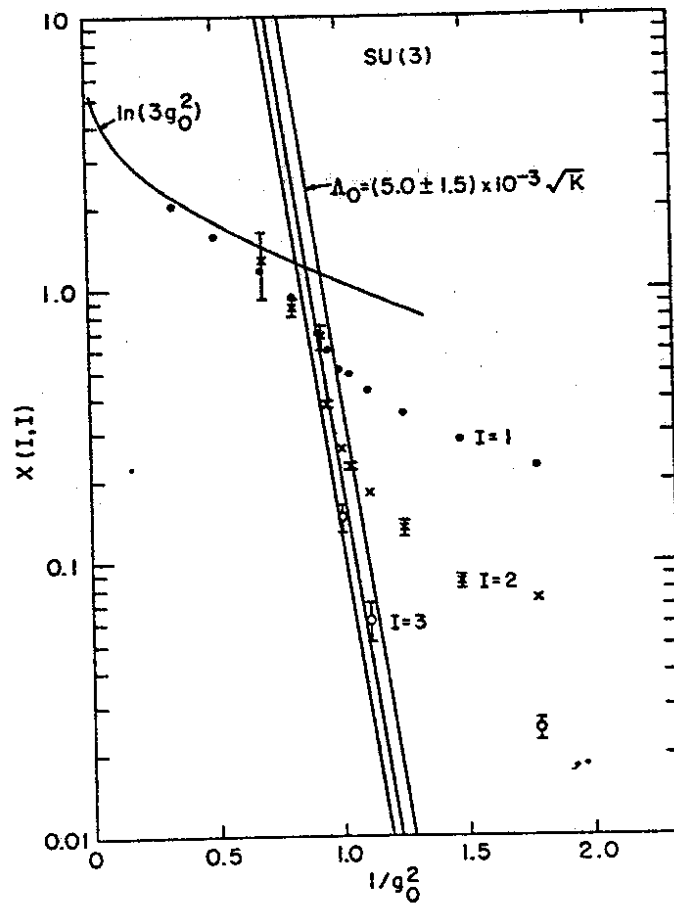


Fig. 1. The effective force $\chi(I,I)$ between quark sources separated by I lattice spacings as a function of the bare charge.

long range constant force K measured in lattice units. Comparing with the asymptotic freedom gives for SU(3)

$$\frac{\Lambda_0}{\sqrt{K}} = (5 \pm 1.5) \times 10^{-3} \quad (5)$$

Using the relation between Λ_0 and the more conventional Λ_{mom}

$$\Lambda_{\text{mom}} = 83.5 \Lambda_0 \quad (6)$$

and using the string model estimate of K from the Regge slope, we find

$$\Lambda_{\text{mom}} = 170 \pm 50 \text{ MeV} \quad (7)$$

This is comfortably close to current phenomenological values;

however, the effects of light quarks are ignored in this calculation.

A second number characterizing the solution of pure gauge theory is the physical temperature at which a deconfining phase transition occurs.⁵ At this temperature space fills with a soup of gluonic flux and it no longer requires an infinite energy to isolate a source in the fundamental representation of the gauge group. Kajantie, Montonen and Pietarinen⁶ have recently studied this transition in Monte Carlo studies of the SU(3) theory and find

$$T_c \sim \Lambda_{\text{mom}} \quad (8)$$

It is somewhat puzzling in this quarkless theory containing no pions that this transition occurs at such a remarkably low temperature, considerably below a typical hadronic mass.

I now turn to a number which has been frustratingly difficult to extract from the Monte Carlo analysis. This is the mass gap or correlation length in the pure glue theory. Thus far attempts to measure this "glueball" mass have been limited to the SU(2) theory. Plagued by statistical errors, early estimates^{1,7} gave

$$m \sim 1.4 \sqrt{K} \quad (9)$$

Indeed, this is one area where the strong coupling expansion approach may beat the Monte Carlo; Münster has quoted⁸ values for both SU(2) and SU(3)

$$\begin{aligned} m &= 1.8 \pm .8 \sqrt{K} && \text{SU}(2) \\ m &= 2.9 \pm .8 \sqrt{K} = 1.3 \pm 0.4 \text{ GeV} && \text{SU}(3) \end{aligned} \quad (10)$$

This latter value is quite acceptable phenomenologically.

I would now like to briefly discuss an ongoing attempt by Brower, Nauenberg and myself to extract this number in a novel way.⁹ The basic idea is that effects of a finite lattice size should fall exponentially with the lattice dimension in units of the correlation length. Thus motivated, we accurately measured¹⁰ the internal energy for various finite lattices up to 10^4 . A straightforward transfer matrix analysis relates the internal energy per plaquette on an N^4 lattice to that on an infinite lattice

$$P(N) = P(\infty) - \frac{\sqrt{2} r}{48\pi\beta_0} \frac{(ma)^{5/2}}{N^{3/2}} e^{-maN} \left(1 + O\left(\frac{1}{maN}\right)\right) \quad (11)$$

Here the only unknowns are the mass m and the factor r , which represents the degeneracy of the first glueball state. A spin s state contributes $2s+1$ to r . Although in principle for a large enough lattice only the lightest state will survive, in practice we should expect finite size effects from a superposition of several low lying states. Indeed, the degeneracy factor suggests that a spin two state may dominate over a lighter spin zero particle.

The observed finite size effects are small, only clearly observable up to about a 6^4 lattice. Using lattices larger or equal to 4^4 sites, the data does not permit determining r and ma separately for each value of coupling. Even though small, the signal is too large for the degeneracy factor in eq. (11) to be unity. Indeed, a simple fit suggests $r \sim 10-30$. The mass value obtained is strongly correlated with the degeneracy, but for r in this range

$$m = (150 \pm 50)A_0 = 1.9 \pm .6 K$$

adequately fits the asymptotic freedom prediction.

We thus conclude that the mass is still hard to measure, but the spectrum must be extremely rich. Note that the bag model also predicts a large number of low lying states.¹¹

These, then, are the three theoretically clear parameters which have been extracted from the Monte Carlo studies. There are, however, an enormous number of practitioners of this art. Most of the lattice work has not emphasized the continuum limit, but artifacts of the lattice theory itself. Indeed, as statistical mechanical systems, lattice gauge theories have been found to have a fascinating and rich phase structure. I will end this lecture by showing two interesting phase diagrams obtained via Monte Carlo techniques.

In fig. 2 I show the phase diagram for a coupled Z_2 Higgs-Gauge system.¹² The parameter β is the inverse gauge coupling and β_H is a

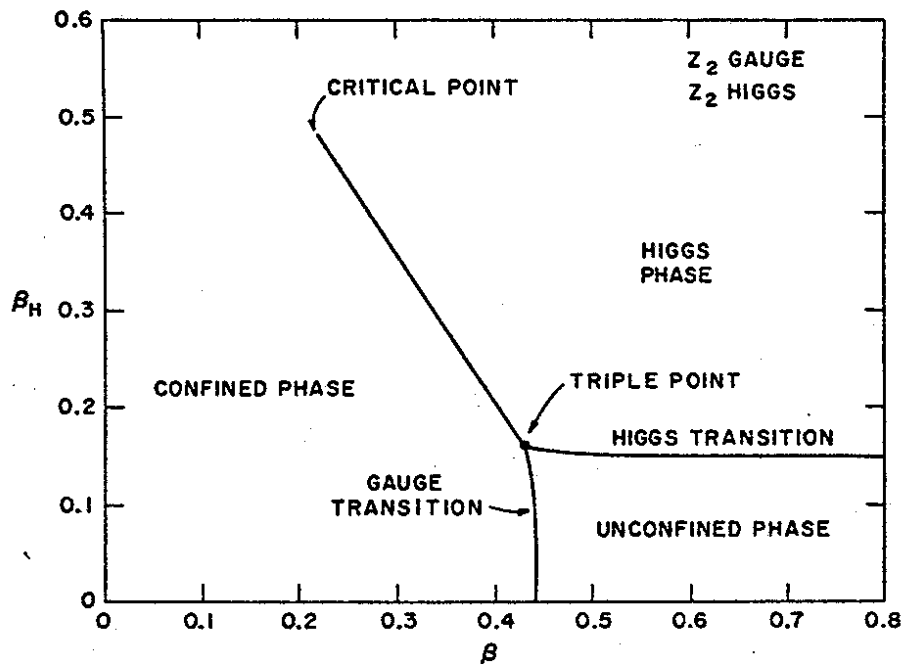


Fig. 2. The phase diagram for the coupled Z_2 spin gauge system.

parameter driving a Higgs mechanism. In this model with small β_H and large β we have a conventional unconfined phase. In a more realistic theory with a continuum gauge group this phase would possess massless gauge bosons. As β_H is increased one leaves this phase through a Higgs transition. Alternatively, at fixed β_H , we can pass through a confining transition at strong gauge coupling as β is decreased. Note, however, that the Higgs and confinement phases are smoothly connected around a critical point. This is an example of a mechanism described by Fradkin and Shenker,¹³ whereby whenever Higgs fields lie in the fundamental representation of the gauge group, the confinement and Higgs phases are qualitatively the same. Thus the standard electroweak theory may have equivalent but superficially disjoint descriptions.

In fig. 3 I show a phase diagram for a pure SU(2) gauge theory but with the lattice action characterized by two parameters.¹⁴ The action per plaquette is

$$S_{\square} = \beta \left(1 - \frac{1}{2} \text{Tr} U_{\square} \right) + \beta_A \left(1 - \frac{1}{3} \text{Tr}_A U_{\square} \right)$$

Here U_{\square} is the group element associated with the plaquette in question, and Tr and Tr_A represent traces taken in the fundamental and adjoint representations respectively. For $\beta_A = 0$ we have the standard Wilson SU(2) theory whereas for $\beta = 0$ we have an SO(3) lattice gauge model. As β_A goes to infinity, the model approaches the pure Z₂ theory.

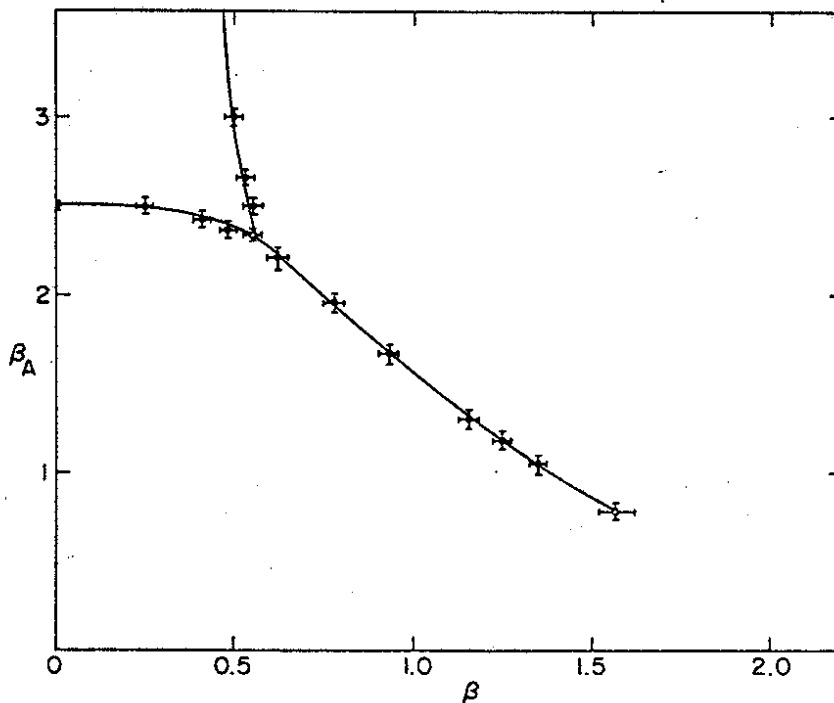


Fig. 3. The phase diagram with a generalized SU(2) action.

The rich structure with its triple point and new critical point demonstrates the naivety of the old lore that non-Abelian gauge theories have no phase transitions. This structure, however, appears to be purely a lattice artifact and is irrelevant to continuum physics. The lattice theory should not be trusted phenomenologically when the lattice spacing becomes comparable to hadronic dimensions.

In the last year unexpected phase transitions¹⁵ have been found for the $SO(3)$ and $SU(N)$ for $N \geq 4$ lattice models. Indeed, $SU(2)$ and $SU(3)$ are the only known groups showing a smooth passage from strong to weak coupling. The $SO(3)$ transition has been described in terms of a monopole condensation.¹⁶ The large N transitions are probably closely related in that the Wilson action has several local minima beyond the vanishing fields of the classical limit. Tunnelling into these minima can generate approximate monopole configurations which may condense as in $SO(3)$. If this is a correct picture, modifying the Wilson action to eliminate the metastability of such configurations should remove the $SU(N)$ transitions. This is currently under investigation.

In conclusion, the past few years have been extremely exciting for the lattice theory. New results have been appearing faster than we can absorb them. Although we have calculated only a few parameters of the continuum theory, these are remarkable numbers indeed. They characterize non-perturbative aspects of the solution of a non-trivial four dimensional field theory.

REFERENCES

1. G. Bhanot and C. Rebbi, Nucl. Phys. B189 [FSZ], 469 (1981).
2. W. E. Caswell, Phys. Rev. Lett. 33, 224 (1974); D. R. T. Jones, Nucl. Phys. B75, 531 (1974).
3. W. Celmaster and R. J. Gonsalves, Phys. Rev. D20, 1420 (1979); A. Hasenfratz and P. Hasenfratz, Phys. Lett. 93B, 165 (1980).
4. M. Creutz, Phys. Rev. Lett. 45, 313 (1980).
5. A. M. Polyakov, Phys. Lett. 72B, 477 (1978); L. Susskind, Phys. Rev. D20, 2610 (1979); L. McLerran and B. Svetitsky, Phys. Lett. 98B, 195 (1981); J. Kuti, J. Polonyi, and K. Szlachanyi, Phys. Lett. 98B, 199 (1981); J. Engels, F. Karsch, H. Satz and I. Montvay, preprint (1980).
6. K. Kajantie, C. Montonen, and E. Pietarinen, preprint (1981).
7. B. Berg, Phys. Lett. 97B, 401 (1980); J. Engels, F. Karsch, H. Satz and I. Montvay, preprint (1981).
8. G. Munster, Nucl. Phys. B190, 439 (1981); erratum (1981).
9. R. Brower, M. Creutz, and M. Nauenberg, in preparation.
10. R. Brower, M. Nauenberg and T. Schalk, preprint (1981).
11. J. Donoghue, talk at this conference.
12. M. Creutz, Phys. Rev. D21, 1006 (1980).
13. E. Fradkin and S. Shenker, Phys. Rev. D19, 3682 (1979).
14. G. Bhanot and M. Creutz, preprint (1981).
15. J. G. Halliday and A. Schwimmer, Phys. Lett. 101B, 327 (1981); J. Greensite and B. Lautrup, Phys. Rev. Lett. 47, 9 (1981); M. Creutz, Phys. Rev. Lett. 46, 1441 (1981).
16. J. G. Halliday and A. Schwimmer, Phys. Lett. 102B, 337 (1981); E. Tomboulis, Phys. Rev. D23, 2371 (1981).

8