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U(3) FOUR-DIMENSIONAL LATTICE GAUGE THEORY AND MONTE CARLO CALCULATIONS

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Monte Carlo results for the pure U(3) lattice gauge theory on a 6^4 lattice are reported. Wilson loops and the string tension are presented The first-order phase transition in U(3) is reflected quite clearly in a discontinuity in the string tension at $\beta = \beta_c$ The U(1) factor of U(3) is extracted using the determinant of the Wilson loops As expected, the U(1) component appears to deconfine at the phase transition

In a previous paper [1], we found a first-order phase transition in pure U(3) gauge theory on a four-dimensional space-time lattice. In order to examine the nature of this transition in more detail, we study the Wilson loops and, hence, the string tension for this gauge group. The U(3) gauge group contains SU(3) and U(1) components which should decouple at low temperature. We should then be left with the confinement of SU(3) color and the deconfinement of U(1) charges. In a recent paper [2] we verified the analogous behavior with the gauge group U(2). This paper extends these results to U(3).

We consider U(N) lattice gauge theory in four dimensions. The lattice spacing is a cutoff for small distances and, hence, the inverse lattice spacing is an ultraviolet cutoff in momentum space. A matrix $U_{ij} \in \mathrm{U}(N)$ is associated with each link joining the nearest-neighbor lattice sites, denoted by i and j. We decompose these matrices in the form

$$U_{ij} = \exp(i\theta_{ij})\overline{U}_{ij},$$

where $\overline{U}_{ij}$ is an $N \times N$ unitary unimodular matrix of SU(N) and θ is an angle

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associated with the compact U(1) gauge degree of freedom. We cover the U(N) gauge group manifold when the angle θ covers the interval $[0, 2\pi/N)$ and $\bar{U}_{ij}$ covers SU(N). By traversing a link in the reverse direction, we get the inverse group element, i.e.,

$$U_{ji} = (U_{ij})^{-1}.$$

The expectation value of the observable $A[U]$ is defined by

$$\langle A(\beta)\rangle = \frac{1}{Z(\beta)} \int \Big(\prod_{\langle i,j\rangle} \mathrm{d}U_{ij} \Big) A[U] \exp(-\beta S[U]),$$

where $Z(\beta)$ is the partition function defined to normalize the expectation value of the operator 1. In the above expression, β is the inverse temperature which is related to the bare coupling constant by $\beta = 2N/g_0^2$ and the measure is the normalized invariant Haar measure for the U(N) gauge group. We define the action S as a sum over all unoriented plaquettes $\Box$ of the lattice

$$S = \Sigma S_\Box = \Sigma\Big(1 - \frac{1}{N}\,\mathrm{Re\,Tr}\, U_\Box\Big),$$

when $U_\Box = U_{ij}U_{jk}U_{kl}U_{li}$ is the parallel transporter around the plaquette $\Box$ with the boundary ij, jk, kl and li. We use periodic boundary conditions throughout our calculations and the standard Monte Carlo method of Metropolis et al. [3, 4] to equilibrate our lattice. From now on we will specialize to the case of $N = 3$, i.e. U(3).

The Wilson loop [5] on the lattice is defined by the expectation value

$$W(I, J) = \tfrac{1}{3}\langle \mathrm{Re\,Tr}\, U_{\mathrm{C}}\rangle,$$

where C is a rectangular contour of dimensions I and J and the product of link variables around the contour C is denoted by U_{C}. The leading-order high-temperature expansion for the Wilson loop is

$$W(I, J) = \left(\tfrac{1}{18}\beta\right)^{IJ} \times \left(1 + \mathrm{O}(\beta^2)\right), \tag{1}$$

while the leading-order low-temperature expansion for the average action per plaquette is [6]

$$\langle E\rangle = 1 - W(1,1) = \frac{9}{4\beta} + \mathrm{O}(\beta^{-2}). \tag{2}$$

For rectangular dimensions large compared to the correlation length, the Wilson loop should assume the asymptotic form

$$W(I, J) = \exp(-A - B \cdot IJ - C \cdot 2(I + J)),$$

where, for convenience, we have set the lattice spacing to 1, and for a given β, the parameters A, B and C are constants. When the asymptotic form of the above equation applies, the string tension B is easily extracted by calculating the quantity [7]

$$\chi(I, J) = -\ln\left[\frac{W(I, J)W(I-1, J-1)}{W(I, J-1)W(I-1, J)}\right].$$

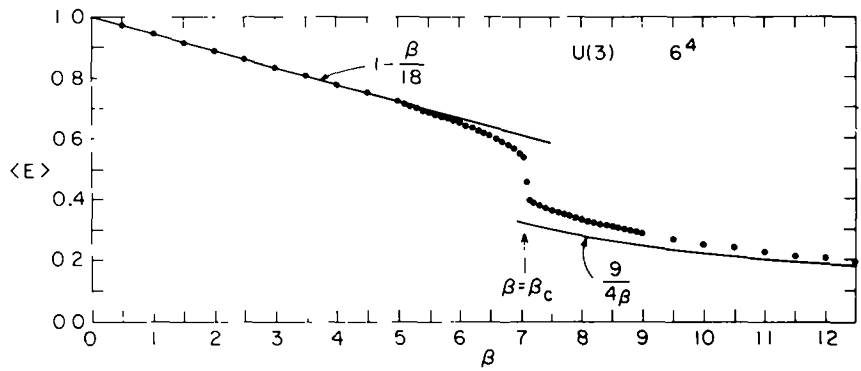

Fig 1 The average action per plaquette $\langle E \rangle$ for pure U(3) gauge theory on a 6^4 lattice as a function of the inverse temperature β The curves represent the leading-order high- and low-temperature expansions of eqs (1) and (2)

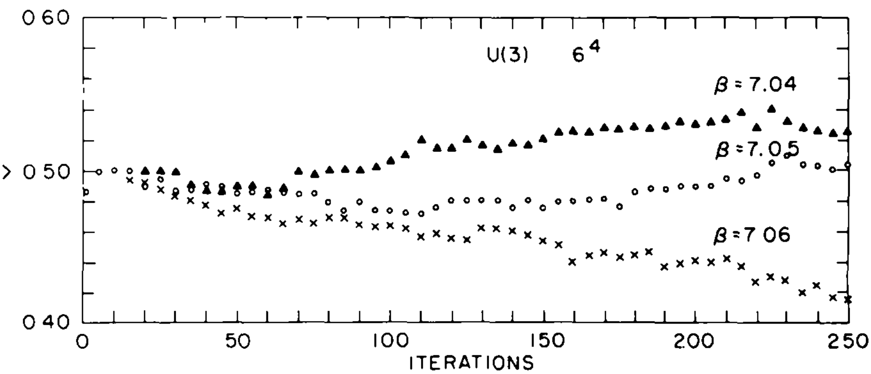

Fig 2 The evolution of the average action per plaquette $\langle E \rangle$ for pure U(3) gauge theory on a 6^4 lattice as a function of the number of iterations through the lattice for mixed phase starting lattices for various values of the inverse temperature β

The leading-order high-temperature expansion for the string tension is

$$\chi(I, J) = -\ln(\tfrac{1}{18}\beta) + O(\beta^2). \tag{3}$$

We can extract the U(1) component of U(3) by calculating the determinant [8] for each loop considered as a 3×3 matrix in U(3) and averaging over all similar loops in each configuration to give the average determinants denoted by

$$\bar{W}(I, J) = \langle \mathrm{DET}(U_c) \rangle .$$

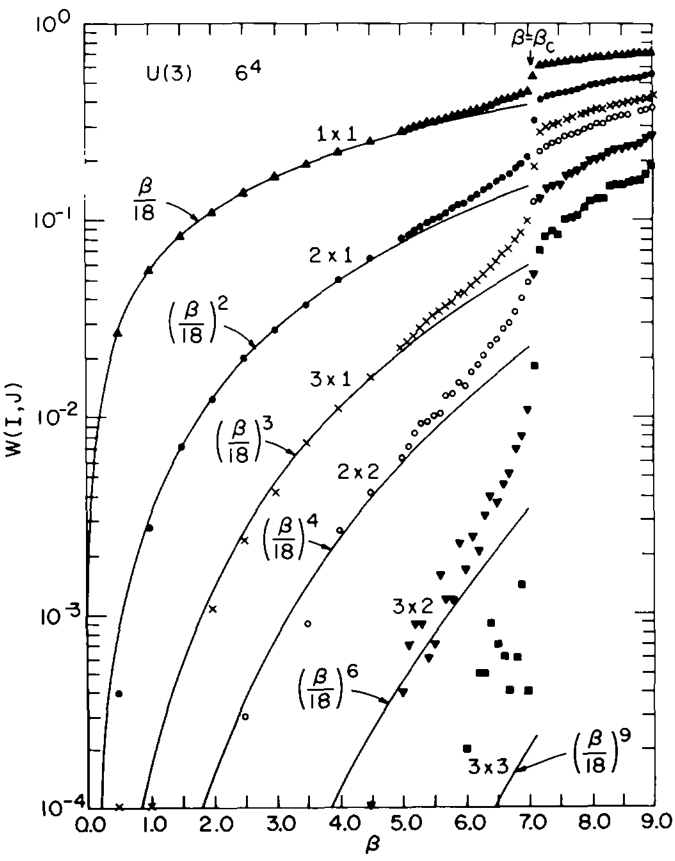

Fig 3 The Wilson loops $W(I, J)$ for pure U(3) gauge theory on a 6^4 lattice as a function of the inverse temperature β. The full upward triangles represent $(I, J) = (1, 1)$, the full circles (2, 1), the crosses (3, 1), the open circles (2, 2), the full downward triangles (3, 2) and the full squares (3, 3) The curves represent the leading-order high-temperature expansion of eq (1)

From these quantities we form the new logarithmic ratio

$$\bar{\chi}(I,J) = -\ln\left[\frac{\overline{W}(I,J)\overline{W}(I-1,J-1)}{\overline{W}(I,J-1)\overline{W}(I-1,J)}\right].$$

If at weak coupling the theory does not confine U(1) charges, then as the loop grows this quantity should go to zero in the low-temperature region. The leading-order high-temperature expansion for the average determinants is

$$\overline{W}(I,J) = \left[\tfrac{1}{1296}\beta^3 + O(\beta^5)\right]^{IJ}, \tag{4}$$

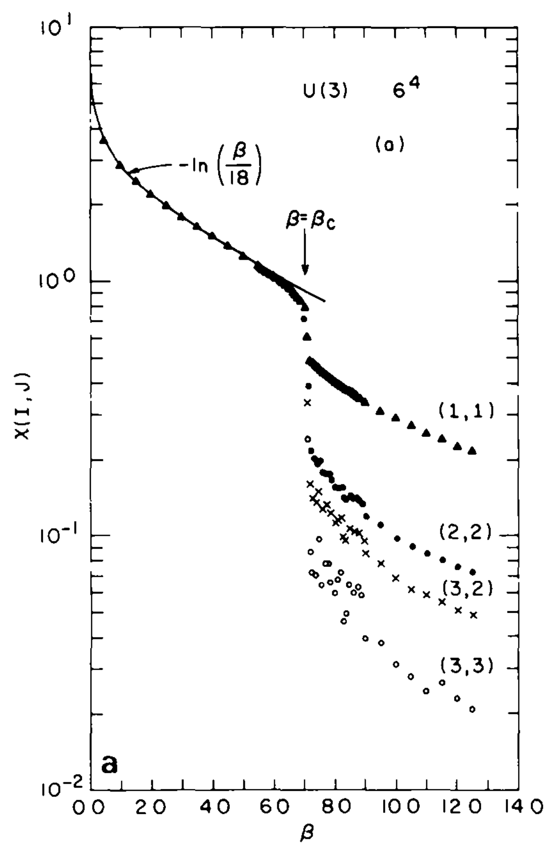

Fig 4 (a, b) The function $\chi(I,J)$ for pure U(3) gauge theory on a 6^4 lattice as a function of the inverse temperature β The full upward triangles represent $(I,J) = (1,1)$, the full circles (2,2), the crosses (3,2) and the open circles (3,3) Also shown in the diagram is the leading-order high-temperature expansion of eq (3)

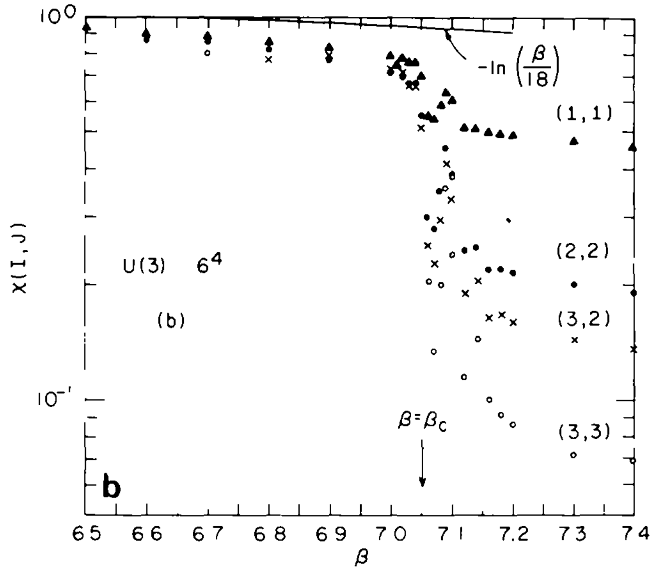

Fig 4 (continued)

and for the quantity $\bar{\chi}(I, J)$ is

$$\bar{\chi}(I, J) = -\ln\left(\tfrac{1}{1296}\beta^3\right) + O(\beta^5). \tag{5}$$

In fig. 1 we show the average action per plaquette for pure U(3) gauge theory as a function of the inverse temperature β on a 6^4 lattice. To obtain these results, we first performed 200 iterations through the lattice with 20 Monte Carlo updates per link. This appears to adequately equilibrate the lattice. We then averaged over the next 100 iterations through the lattice. We used disordered starts for $\beta \leqslant 5.5$, mixed phase [9] starts for $5.6 < \beta < 9.0$ and ordered starts for $\beta \geqslant 9.0$. Mixed phase runs were used in the crossover region in order to avoid problems of supercooling or superheating associated with disordered and ordered starts, respectively. In the mixed phase starts, the fourth axis of the euclidean lattice, the time axis, was divided in two with the upper half of the link variables disordered and the lower half ordered. In fig. 1 the leading-order high- and low-temperature expansions of eqs. (1) and (2), respectively, are also shown. In fig. 2 we show some mixed phase runs for the average action per plaquette in the vicinity of the critical inverse temperature and we see evidence for a first-order phase transition at the inverse temperature of $\beta_c = 7.06 \pm 0.01$. The Wilson loops of size up to 3×3 are presented in fig. 3. Eq. (1) accurately describes the strong coupling behavior.

In fig. 4a we present the quantity $\chi(I, J)$, for $(I, J) = (1,1), (2,2), (3,2)$ and $(3,3)$ as a function of the inverse temperature β. Also shown in fig. 4 is the high-temperature expansion of eq. (3) which agrees well with the Monte Carlo data for $\beta < 7$. In fig. 4b we show the vicinity of the critical inverse temperature in some detail to exhibit the sharp jump, presumably a discontinuity, in the string tension.

At large β, the U(1) part of our matrices should decouple and leave an effective SU(3) theory. In fig. 5 we present the χ ratios for both U(3) and SU(3) gauge theories. The U(3) results mimic SU(3) results shifted by approximately 1.1 units in β. This shift may be calculable perturbatively.

In fig. 6 we show the average U(1) action per plaquette $\langle \bar{E} \rangle = 1 - \bar{W}(1,1)$ as a function of the inverse temperature β on a 6^4 lattice. The leading-order high-temper-

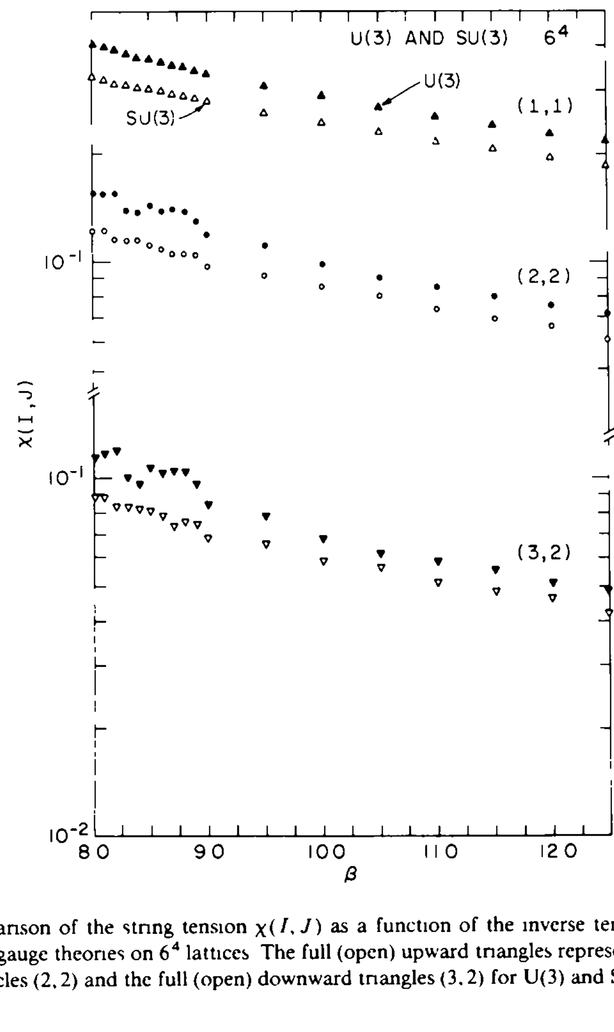

Fig 5 A comparison of the string tension $\chi(I, J)$ as a function of the inverse temperature β for pure U(3) and SU(3) gauge theories on 6^4 lattices The full (open) upward triangles represent $(I, J) = (1,1)$, the full (open) circles (2,2) and the full (open) downward triangles (3,2) for U(3) and SU(3), respectively

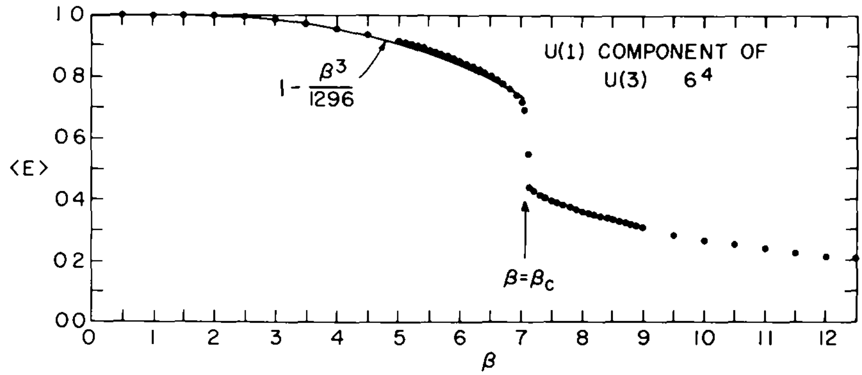

Fig 6 The average U(1) action per plaquette $\langle \bar{E} \rangle$ on a 6^4 lattice as a function of the inverse temperature β The curve represents the leading-order high-temperature expansion of eq (4)

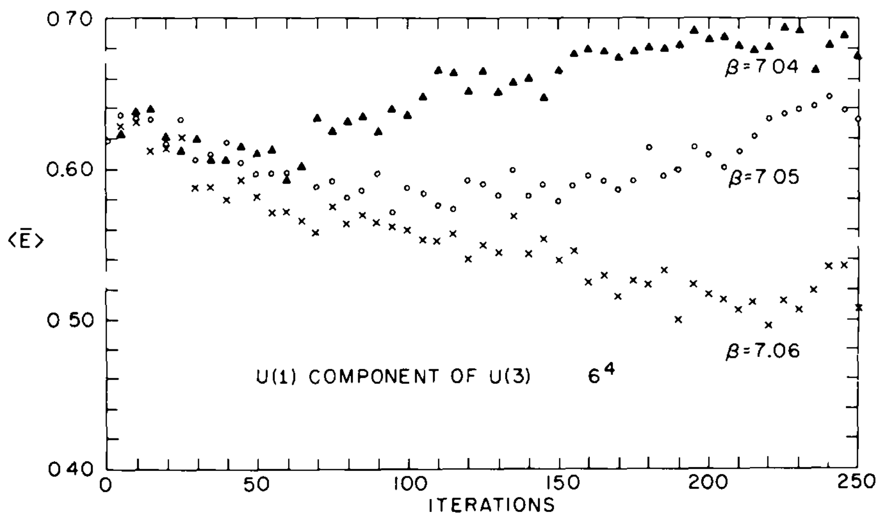

Fig 7 The evolution of the average U(1) action per plaquette $\langle \bar{E} \rangle$ on a 6^4 lattice as a function of the number of iterations through the lattice for mixed phase starting lattices for various values of the inverse temperature β

ature expansion of eq. (4) is also shown. In fig. 7 some mixed phase runs for $\langle \bar{E} \rangle$ near the critical inverse temperature are shown. The first-order nature of the transition is also quite clear in this quantity; this contrasts with the second-order transition in the pure U(1) model [10]. The U(1) Wilson loops $\overline{W}(I, J)$ of size up to 3×3 are shown in fig. 8 with eq. (4) shown for comparison.

In fig. 9 we show the logarithmic ratios $\bar{\chi}(I, J)$ for $(I, J) = (1,1), (2,2), (3,2)$ and $(3,3)$ as a function of the inverse temperature β. We also indicate the leading-order high-temperature expansion of eq. (5). In the low-temperature region, these quantities decrease with loop size faster than the quantities $\chi(I, J)$ shown in fig. 4. Indeed, the expectation is that the quantities $\bar{\chi}(I, J)$ should go to zero in the low-temperature region as the loop size increases. Thus, the determinant of the loops would appear not to confine U(1) charges in the low-temperature region as in the pure U(1) gauge theory [10, 11].

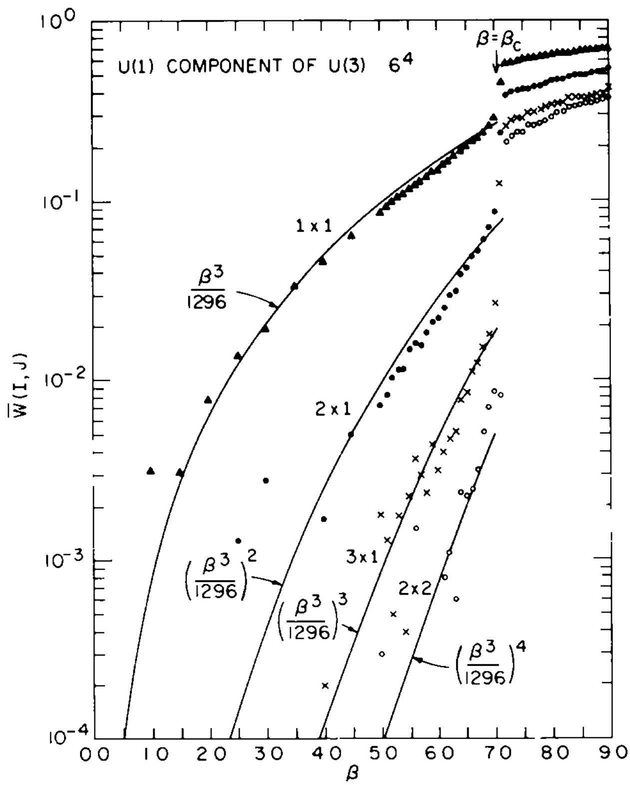

Fig 8 The Wilson loops $\overline{W}(I, J)$ for the U(1) component of pure U(3) gauge theory on a 6^4 lattice as a function of the inverse temperature β The full upward triangles represent $(I, J) = (1, 1)$, the full circles (2, 1), the crosses (3, 1) and the open circles (2, 2) The curves represent the leading-order high-temperature expansion of eq (4)

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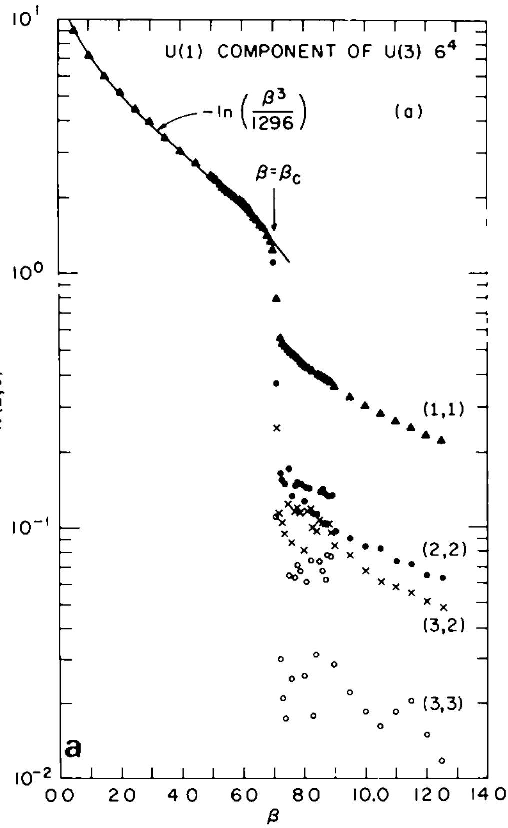

Fig 9 The string tension $\bar{\chi}(I,J)$ for the U(1) component of pure U(3) gauge theory on a 6^4 lattice as a function of the inverse temperature β. The full upward triangles represent $(I,J)=(1,1)$, the full circles (2,2), the crosses (3,2) and the open circles (3,3) Also shown in the diagram is the leading-order high-temperature expansion of eq (5)

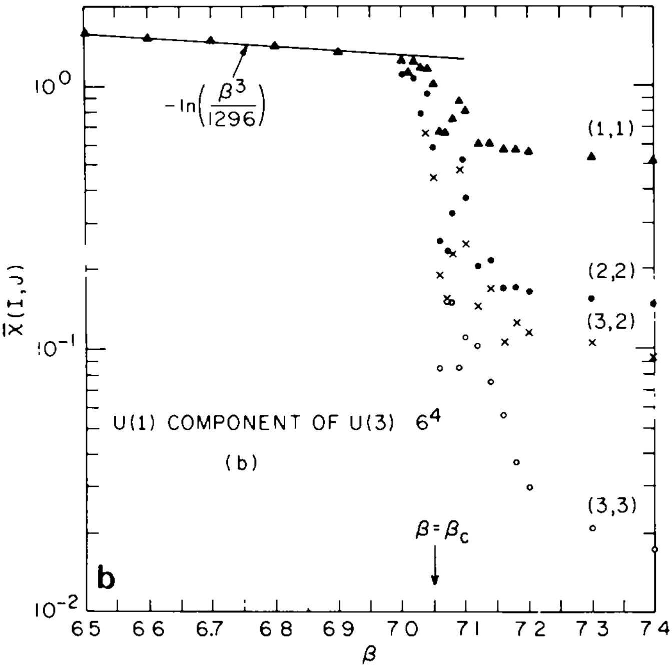

Fig 9 (continued)

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