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Species Doubling and Effective Lagrangians

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Coupling gauge fields to the chiral currents from an effective Lagrangian for pseudoscalar mesons naturally gives rise to a species doubling phenomenon similar to that seen with fermionic fields in lattice gauge theory.

Species doubling is deeply entwined with the famous axial anomalies. A lattice regulator removes all infinities; so, anomalies cannot appear without explicit symmetry breaking. As the regulator is removed, a remnant remains determining an overall chiral phase. This phase is the well known strong CP parameter θ [2]. Most lattice schemes also mutilate non-singlet chiral symmetries, although ways around this exist [1].

For the gauge interactions of the electroweak theory, non-perturbative chiral issues remain unresolved. The W bosons couple in a parity violating manner, and rely for consistency on a subtle anomaly cancellation between quarks and leptons. While a flurry of recent work treats fermions with a separate limit [4–6], it remains unclear how to implement this cancellation in a fully finite and gauge-invariant manner.

We argue that the doubling problem is not unique to the lattice approach, but is a general consequence of chiral anomalies. This presentation is based on our recent letter [3]. Starting with an effective Lagrangian for the pseudoscalar mesons, we couple gauge fields to the chiral currents. When these fields are chiral, the process naturally introduces additional particles mirroring the original theory. As with lattice doublers, the mirror fields cancel anomalies.

In a chiral Lagrangian approach, the effects of anomalies appear in a term introduced by Wess and Zumino [7], and later elucidated by Witten [8]. This involves extending the fields into an internal space, only the boundary of which is relevant to the equations of motion. On coupling to a local gauge field, the boundary can acquire additional contributions. We argue that these are most naturally written in terms of doubler fields.

We start with quark fields $\psi^a(x)$ interacting with non-Abelian gluons. We suppress all indices except flavor, represented by the index a, and space-time, represented by x. From ψ we project right and left handed parts, $\psi_R^a = \frac{1}{2}(1+\gamma_5)\psi^a$ and $\psi_L^a = \frac{1}{2}(1-\gamma_5)\psi^a$. We ignore fermion masses.

This theory with massless quarks is invariant under a global $SU(n_f)\times SU(n_f)$ symmetry, where n_f represents the number of flavors. The quark fields transform as $\psi_L^a \to \psi_L^b g_L^{ba}$ and $\psi_R^a \to \psi_R^b g_R^{ba}$. Here g_L and g_R are elements of $SU(n_f)$.

The chiral symmetry is spontaneously broken by the æther, resulting in n_f^2-1 Goldstone bosons and a remaining explicit $SU(n_f)$ flavor symmetry. The composite field $\overline{\psi}_R^a\psi_L^b$ acquires an expectation value. Chiral symmetry allows chosing a standard æther with, say, $\langle\overline{\psi}_R^a\psi_L^b\rangle = v\delta^{ab}$. Here the parameter v determining the magnitude of the expectation value requires a renormalization scheme for precise definition. The æther is degenerate, and one could choose to replace δ^{ab} by an arbitrary element g^{ab} of $SU(n_f)$. The basic idea of the effective Lagrangian is to promote this element into a local field $g(x)$. Slow variations of this field represent the Goldstone bosons. The chiral symmetry is an invariance under $g(x) \to g_L^\dagger g(x) g_R$.

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The effective Lagrangian approach represents an expansion in light particle momenta [9]. The lowest order action is

$$S_0 = \frac{F_\pi^2}{4}\int d^4x\ \mathrm{Tr}(\partial_\mu g \partial_\mu g^\dagger). \qquad (1)$$

The constant F_π has an experimental value 93 MeV. In terms of conventional fields, $g = \exp(i\pi\cdot\lambda/F_\pi)$, where the $n_f^2 - 1$ matrices λ generate $SU(n_f)$ and are normalized $\mathrm{Tr}\lambda^\alpha\lambda^\beta = 2\delta^{\alpha\beta}$.

From this action the equations of motion are $\partial_\mu J^\alpha_{L,\mu} = 0$, where the "left" current is

$$J^\alpha_{L,\mu} = \frac{iF_\pi^2}{4}\mathrm{Tr}\lambda^\alpha(\partial_\mu g)g^\dagger \qquad (2)$$

Equivalently, one can work with "right" currents. There is a vast literature on adding higher derivative terms [9].

We are interested in a special higher derivative coupling describing the effects of anomalies. This term cannot be written as an integral of a local expression in $g(x)$, even though the resulting contribution to the equations of motion is fully local [7,8,10]. Continuing to write the equations of motion in terms of a divergence free current, a possible addition which satisfies the required symmetries is

$$\begin{aligned} J^\alpha_{L,\mu} = & \frac{iF_\pi^2}{4}\mathrm{Tr}\lambda^\alpha(\partial_\mu g)g^\dagger \\ & + \frac{in_c}{48\pi^2}\epsilon_{\mu\nu\rho\sigma}\mathrm{Tr}\lambda^\alpha(\partial_\nu g)g^\dagger(\partial_\rho g)g^\dagger(\partial_\sigma g)g^\dagger \end{aligned} \qquad (3)$$

To obtain an action generating the above requires extending $g(x)$ beyond a simple mapping of space-time into the group. We introduce an auxiliary variable s to interpolate between the field $g(x)$ and some fixed group element g_0. Thus consider $h(x,s)$ satisfying $h(x,1) = g(x)$ and $h(x,0) = g_0$. This extension is not unique, but the equations of motion are independent of the chosen path. We now write

$$\begin{aligned} S = & \frac{F_\pi^2}{4}\int d^4x\ \mathrm{Tr}(\partial_\mu g\partial_\mu g^\dagger) \\ & + \frac{n_c}{240\pi^2}\int d^4x\int_0^1 ds\ \epsilon_{\alpha\beta\gamma\delta\rho}\mathrm{Tr}h_\alpha h_\beta h_\gamma h_\delta h_\rho. \end{aligned} \qquad (4)$$

Here we define $h_\alpha = i(\partial_\alpha h)h^\dagger$ and regard s as a fifth coordinate.

For equations of motion, consider a small variation of $h(x,s)$. This changes the final integrand by a total divergence, which then integrates to a surface term. Working with either spherical or toroidal boundary conditions in the space-time directions, this surface only involves the boundaries of the s integration. When $s = 0$, space-time derivatives acting on the constant matrix g_0 vanish. The surface at $s = 1$ generates precisely the desired additional term in Eq. (3).

The last term in Eq. (4) represents a piece cut from the S_5 sphere appearing in the structure of $SU(n_f)$ for $n_f \geq 3$. The mapping of four dimensional space-time into the group surrounds this volume. Chiral rotations shift this region around, leaving its measure invariant. As emphasized by Witten [8], this term is ambiguous. Different extensions into the s coordinate can modify the above five dimensional integral by an integer multiple of $480\pi^3$. To have a well defined quantum theory, the action must be determined up to a multiple of 2π. Thus the quantization of n_c to an integer, the number of "colors."

Crucial here is the irrelevance of the starting group element g_0 at the lower end of the s integration. Our main point is the difficulty of maintaining this condition when we make the chiral symmetry local. As usual, this requires the introduction of gauge fields. Under the transformation $g(x) \to g_L^\dagger(x)g(x)g_R(x)$, derivatives of g transform as

$$\partial_\mu g \longrightarrow g_L^\dagger\left(\partial_\mu g - \partial_\mu g_L g_L^\dagger g + g\partial_\mu g_R g_R^\dagger\right)g_R \qquad (5)$$

To compensate, we introduce left and right gauge fields transforming as

$$\begin{aligned} A_{L,\mu} &\longrightarrow g_L^\dagger A_{L,\mu}g_L + ig_L^\dagger\partial_\mu g_L \\ A_{R,\mu} &\longrightarrow g_R^\dagger A_{R,\mu}g_R + ig_R^\dagger\partial_\mu g_R \end{aligned} \qquad (6)$$

Then the combination

$$D_\mu g = \partial_\mu g - iA_{L,\mu}g + igA_{R,\mu} \qquad (7)$$

transforms nicely: $D_\mu g \to g_L^\dagger D_\mu g g_R$. Making the generalized minimal replacement $\partial_\mu g \to D_\mu g$ in S_0, we find a gauge invariant action.

A problem arises when we go on to the Wess-Zumino term. We require a prescription for the gauge transformation on the interpolated group element $h(x,s)$. Here we note a striking analogy with the domain wall approach to chiral fermions first promoted by Kaplan [11]. There an extra dimension was also introduced, with the fermions being surface modes bound to a four dimensional interface. The usual approach to adding gauge fields involves, first, not giving the gauge fields a dependence on the extra coordinate, and, second, forcing the component of the gauge field pointing in the extra dimension to vanish [12,13]. In terms of a five dimensional gauge field, we take $A_\mu(x,s) = A_\mu(x)$ and $A_s = 0$ for both the left and right handed parts. Relaxing either of these would introduce unwanted degrees of freedom. The natural extension of the gauge transformation is to take $h(x,s) \to g_L^\dagger(x) h(x,s) g_R(x)$ with $g_{L,R}$ independent of s.

We now replace the derivatives in the Wess-Zumino term with covariant derivatives. This alone does not give equations of motion independent of the interpolation into the extra dimension. However, adding terms linear and quadratic in the gauge field strengths allows construction of a five dimensional Wess-Zumino term for which variations are again a total derivative. This gives

$$S_{WZ} = \frac{n_c}{240\pi^2} \int d^4x \int_0^1 ds\ \Gamma \tag{8}$$

where

$$\begin{aligned} \Gamma = \Gamma_0 + \frac{5i}{2}(i\Gamma_L + i\Gamma_R \\ - \Gamma_{LL} - \Gamma_{RR} - \alpha\Gamma_{LR} - (1-\alpha)\Gamma_{RL}), \end{aligned} \tag{9}$$

α is a free parameter, and

$$\begin{aligned} \Gamma_0 &= \epsilon_{\mu\nu\rho\lambda\sigma} \mathrm{Tr} D_\mu h h^\dagger D_\nu h h^\dagger D_\rho h h^\dagger D_\lambda h h^\dagger D_\sigma h h^\dagger \\ \Gamma_L &= \epsilon_{\mu\nu\rho\lambda\sigma} \mathrm{Tr} D_\mu h h^\dagger D_\nu h h^\dagger D_\rho h h^\dagger F_{L,\lambda\sigma} \\ \Gamma_R &= \epsilon_{\mu\nu\rho\lambda\sigma} \mathrm{Tr} D_\mu h h^\dagger D_\nu h h^\dagger D_\rho h F_{R,\lambda\sigma} h^\dagger \\ \Gamma_{LL} &= \epsilon_{\mu\nu\rho\lambda\sigma} \mathrm{Tr} D_\mu h h^\dagger F_{L,\nu\rho} F_{L,\lambda\sigma} \\ \Gamma_{RR} &= \epsilon_{\mu\nu\rho\lambda\sigma} \mathrm{Tr} D_\mu h F_{R,\nu\rho} F_{R,\lambda\sigma} h^\dagger \\ \Gamma_{RL} &= \epsilon_{\mu\nu\rho\lambda\sigma} \mathrm{Tr} D_\mu h F_{R,\nu\rho} h^\dagger F_{L,\lambda\sigma} \\ \Gamma_{LR} &= \epsilon_{\mu\nu\rho\lambda\sigma} \mathrm{Tr} D_\mu h h^\dagger F_{L,\nu\rho} h F_{R,\lambda\sigma} h^\dagger. \end{aligned} \tag{10}$$

The covariantly transforming field strengths are

$$\begin{aligned} F_{L,\mu\nu} &= \partial_\mu A_{L,\nu} - \partial_\nu A_{L,\mu} - i[A_{L,\mu}, A_{L,\nu}] \\ F_{R,\mu\nu} &= \partial_\mu A_{R,\nu} - \partial_\nu A_{R,\mu} - i[A_{R,\mu}, A_{R,\nu}]. \end{aligned} \tag{11}$$

For the photon, parity invariance fixes $\alpha = 1/2$. The last four terms contain the process $\pi \to 2\gamma$.

This procedure works well for a vector-like gauge field, where we take $g_L(x) = g_R(x)$ and $A_L = A_R$. We could, for example, take g_0 to be the identity, and then the gauge transformation cancels out at $s = 0$. However, difficulties arise on coupling a gauge field to an axial current. Then $g_0 \to g_L^\dagger(x) g_0 g_R(x)$ in general will no longer be a constant group element. After a gauge transformation, variations of the action give new non-vanishing contributions to the equations of motion from the lower end of the s integration.

The simplest solution makes the $s = 0$ fields dynamical. Thus we replace the field $g(x)$ with two fields $g_0(x)$ and $g_1(x)$. The interpolating field now has the properties $h(x,0) = g_0(x)$ and $h(x,1) = g_1(x)$. The action becomes

$$\frac{F_\pi^2}{4} \int d^4x\ \mathrm{Tr}(D_\mu g_0 D_\mu g_0^\dagger + D_\mu g_1 D_\mu g_1^\dagger) + S_{WZ}. \tag{12}$$

While now gauge invariant, the theory differs from the starting model through a doubling of meson species. The extra particles are associated with the second set of group valued fields $g_0(x)$. The Wess-Zumino term of the new fields has the opposite sign since it comes from the lower end of the s integration. Thus, these "mirror" particles have reflected chiral properties and implement a cancellation of all anomalies. In essence, we have circumvented the subtleties in gauging the model. The value of F_π need not be the same for g_0 and g_1; so, their strong interactions might differ in scale. Nevertheless, coupling with equal magnitude to the gauge bosons, the new fields cannot be ignored.

For vector currents, we can remove the doublers using a diagonal mass term at $s = 0$. For example, with a term $M \mathrm{Tr} g_0(x)$ added to the Lagrangian density, M could be arbitrarily large, forcing g_0 towards the identity.

The doublers arise in analogy to the problems appearing in the surface mode approach to chiral lattice fermions [11–14]. In both cases, an extension to an extra dimension is introduced. Difficulties arise from the appearance of an extra interface. This new surface couples with equal strength to the gauge fields.

If we let $g_{L,R}$ depend on s, we expect problems similar to those seen with domain wall fermions. When the gauge fields vary in the extra dimension, four dimensional gauge invariance is lost. Symmetry can be restored via a Higgs field, but this introduces the possibility of unwanted degrees of freedom in the physical spectrum. Ref.[13] explores the possibility of sharply truncating the gauge field at an intermediate value of the extra coordinate. This gives rise to new low energy bound states acting much like the undesired doubler states.

A Higgs field does permit different masses for the extra species. In particular, the matter couplings to the Higgs field can depend on s. Qualitative arguments suggest that triviality effects on such couplings limit their strength, precluding masses for the extra species beyond a typical weak interaction scale. Presumably such constraints are strongest when the anomalies in the undoubled sector are not canceled. With domain-wall fermions, taking a Higgs-fermion coupling to infinity on one wall introduces a plethora of new low energy bound states [14].

These problems emphasize the subtle way anomalies cancel between quarks and leptons. If the contributions of the leptons are ignored, no non-perturbative approach can be expected to accommodate gauged weak currents. When the required cancellations occur between different fermion representations, perturbation theory appears to be consistent, while all known non-perturbative approaches remain awkward.

There are several possible solutions. Mirror particles might exist, perhaps with masses at the weak scale [15]. Such might even be useful in the spontaneous breaking of the electroweak theory [16]. A related alternative involves spontaneous breaking of an underlying vector-like theory containing additional heavy bosons coupling with opposite parity fermions [17]. All of these involve a profusion of new particles awaiting discovery. A speculative solution would twist the extra dimension so that the doubling particles could be among those already observed. This requires the interpolation in the extra dimension to mix the quarks and the leptons, all of which are involved in the anomaly cancellations.

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