

Wilson Fermions at finite temperature

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Abstract

I conjecture on the phase structure expected for lattice gauge theory with two flavors of Wilson fermions, concentrating on large values of the hopping parameter. Numerous phases are expected, including the conventional confinement and deconfinement phases, as well as an Aoki phase with spontaneous breaking of flavor and parity and a large hopping phase corresponding to negative quark masses.

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In this talk I conjecture on the rather rich phase structure expected for lattice gauge theory with Wilson fermions, paying particular attention to what happens for large hopping parameter. I consider both zero and non-zero temperature. I restrict myself to the standard hadronic gauge theory of quarks interacting with non-Abelian gluons. I leave aside issues related to electromagnetism and weak interactions, both of which also raise fascinating issues for lattice field theory.

The parameters of the strong interactions are the quark masses. I implicitly include here the strong CP violating parameter θ , as this can generally be rotated into the mass matrix [1]. The quark masses are in fact the only parameters of hadronic physics, the strong coupling being absorbed into the units of measurement via the phenomenon of dimensional transmutation [2].

For the purposes of this talk, I take degenerate quarks at $\theta = 0$; so, I can consider only a single mass parameter m . I discuss only the two flavor case, as this will make some of the chiral symmetry issues simpler. I also will treat the theory at finite temperature, T , introducing another variable. Finally, as this is a lattice talk, I introduce the lattice spacing a as a third parameter.

On the lattice with Wilson fermions the three parameters (m, T, a) are usually replaced with β , representing the inverse bare lattice coupling squared, the fermion hopping parameter K , and the number of time slices N_t . The mapping between (m, T, a) and (β, K, N_t) is non-linear, well known, and not the subject of this talk.

Note that in considering the structure of the theory in either of these sets of variables, I am inherently talking about finite lattice spacing a . Thus this entire talk is about lattice artifacts.

I start with the (β, K) plane at zero temperature, and defer how this is modified at finite temperature. The β axis with $K = 0$ represents the pure gauge theory of glueballs. This is expected to be confining without any singularities at finite β . The line of varying K with $\beta = \infty$ represents free Wilson fermions [3]. Here, with conventional normalizations, the point $K = \frac{1}{8}$ is where the mass gap vanishes and a massless fermion should appear in the continuum limit. The full interacting continuum limit should be obtained by approaching this point from the interior of the (β, K) plane.

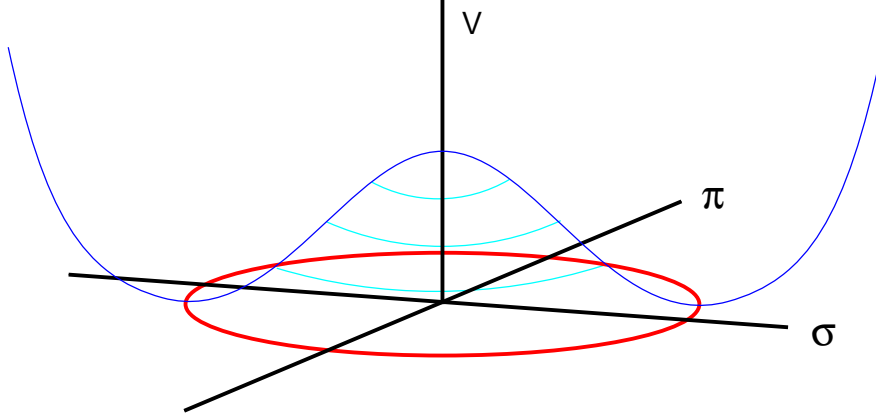


Fig. (1) The effective potential in the canonical sigma model.

While receiving the most attention, this point $K = \frac{1}{8}$ is not the only place where free Wilson fermions lose their mass gap. At $K = \frac{1}{4}$ four doubler species become massless. Also formally at $K = \infty$ six more doublers lose their mass. (Actually, a more natural variable is $\frac{1}{K}$.) The remaining doublers occur at negative K .

The K axis at vanishing β also has a critical point where the confining spectrum appears to develop massless states. Strong coupling arguments as well as numerical experiments place this point somewhere near $K = \frac{1}{4}$, but this is probably not exact. The conventional picture connects this point to $(\beta = \infty, K = \frac{1}{8})$ by a phase transition line representing the lattice version of the chiral limit.

Now I move ever so slightly inside the (β, K) plane from the point $(\infty, \frac{1}{8})$. This should take us from free quarks to a confining theory, with mesons, baryons, and glueballs being the physical states. Furthermore, when the quark is massless, we should have chiral symmetry. Considering here the two flavor case, this symmetry is nicely exemplified in a so called “sigma” model, with three pion fields and one sigma field rotating amongst themselves. Defining the fields

$$\begin{aligned}\sigma &= \bar{\psi}\psi \\ \vec{\pi} &= i\bar{\psi}\gamma_5\vec{\tau}\psi\end{aligned}\tag{1}$$

I consider constructing an effective potential. For massless quarks this is expected to have the canonical sombrero shape stereotyped by

$$V \sim \lambda(\sigma^2 + \vec{\pi}^2 - v^2)^2\tag{2}$$

and illustrated schematically in Fig. (1). The normal æther is taken with an expectation value for the sigma field $\langle\sigma\rangle\sim v$. The physical pions are massless goldstone bosons associated with slow fluctuations of the æther along the degenerate minima of this potential.

As I move up and down in K from the massless case near $\frac{1}{8}$, this effective potential will tilt in the standard way, with the sign of $\langle\sigma\rangle$ being appropriately determined. The role of the quark mass is played by the distance from the critical hopping, $m_q\sim K_c-K$ with $K_c\sim\frac{1}{8}$. At the chiral point there occurs a phase transition, of first order because the sign of $\langle\sigma\rangle$ jumps discontinuously. At the transition point there are massless goldstone pions representing the spontaneous symmetry breaking. With an even number of flavors the basic physics on each side of the transition is the same, since the sign of the mass term is a convention reversible via a chiral rotation. For an odd number of flavors the sign of the mass is significant because the required rotation involves the $U(1)$ anomaly and is not a good symmetry. This is discussed in some detail in my recent paper, Ref. [1]. For the present discussion I stick with two flavors.

A similar picture should also occur near $K=\frac{1}{4}$, representing the point where a subset of the fermion doublers become massless. Thus another phase transition should enter the diagram at $K=\frac{1}{8}$. Similar lines will enter at negative K and further complexity occurs at $K=\infty$. For simplicity, let me concentrate only on the lines from $K=\frac{1}{8}$ and $\frac{1}{4}$.

Now I delve a bit deeper into the (β, K) plane. The next observation is that the Wilson term separating the doublers is explicitly not chiral invariant. This should damage the beautiful symmetry of our sombrero. The first effect expected is a general tilting of the potential. This represents an additive renormalization of the fermion mass, and appears as a beta dependent motion of the critical hopping away from $\frac{1}{8}$. Define $K_c(\beta)$ as the first singular place in the phase diagram for increasing K at given β . This gives a curve which presumably starts near $K=\frac{1}{4}$ at $\beta=0$ and ends up at $\frac{1}{8}$ for infinite β .

Up to this point I have only reviewed standard lore. Now I continue to delve yet further away from the continuum chiral point at $(\beta, K)=(\infty, \frac{1}{8})$. Then I expect the chiral symmetry breaking of the Wilson term to increase and become more than a simple tilting of the Mexican hat. I'm not sure to what extent a multipole analysis of this breaking makes sense, but let me presume that the next effect is a quadratic warping of our sombrero, i.e.

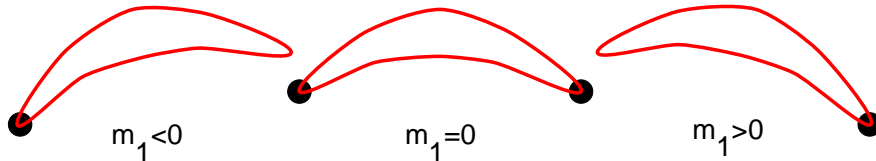


Fig. (2) The effect of a downward warping of the effective potential. The curve represents the warped bottom of the sombrero potential. Here m_1 represents the distance from the critical point. The solid circles represent possible states of the æther. The phase transition now occurs without a diverging correlation length.

a term something like $\alpha\sigma^2$ appearing in the effective sigma model potential. This warping cannot be removed by a simple mass renormalization.

There are two possibilities. This warping could be upward or downward in the σ direction. Indeed, which possibility occurs can depend on the value of β .

Consider first the case where the warping is downward, stabilizing the sigma direction for the æther. At the first order chiral transition, this distortion gives the pions a small mass. The transition then occurs without a diverging correlation length. As before, the condensate $\langle\sigma\rangle$ jumps discontinuously, changing its sign. However, the conventional approach of extrapolating the pion mass to zero from measurements at smaller hopping parameter will no longer yield the correct critical line. The effect of this warping on the potential is illustrated in Fig. (2).

A second possibility is for the warping to be in the opposite direction, destabilizing the σ direction. In this case we expect two distinct phase transitions to occur as K passes through the critical region. For small hopping we have our tilted potential with σ having a positive expectation. As K increases, this tilting will eventually be insufficient to overcome the destabilizing influence of the warping. At a critical point, most likely second order, it will become energetically favorable for the pion field to acquire an expectation value, such a case being stabilized by the upward warping in the sigma direction. As K continues to increase, a second transition should appear where the tilting of the potential is again sufficiently strong to give only sigma an expectation, but now in the negative direction. The effect of this upward warping on the effective potential is illustrated in Fig. (3).

Thus we expect our critical line to split into two, with a rather interesting phase

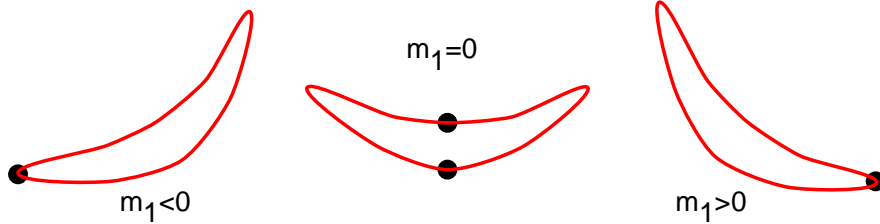


Fig. (3) The effect of an upward warping of the effective potential. Here m_1 represents the distance from $K_c(\beta)$. Now there are two phase transitions, with the intermediate phase having an expectation for the pion field.

between them. This phase has a non-vanishing expectation value for the pion field. As the latter carries flavor and odd parity, both are spontaneously broken. Furthermore, since flavor is still an exact continuous global symmetry, when it is broken Goldstone bosons will appear. In this two flavor case, there are precisely two such massless excitations. If the transitions are indeed second order, a third massless particle appears just at the transition lines, and these three particles are the remnants of the three pions from the continuum theory. This picture of a parity and flavor breaking phase was proposed some time ago by Aoki [4], who presented evidence for its existence in the strong coupling regime. This phase should be “pinched” between the two transitions, and become of less importance as β increases. Whether the phase might be squeezed out at a finite β to the above first order case, or whether it only disappears in the infinite β limit is a dynamical question as yet unresolved.

A similar critical line splitting to give a broken flavor phase should also enter our phase diagram from $(\beta, K) = (\infty, \frac{1}{4})$, representing the first set of doublers. Evidence from toy models [5] is that after this line splits, the lower half joins up with the upper curve from the $(\beta, K) = (\infty, \frac{1}{8})$ point. In these models, there appears to be only one broken parity phase at strong coupling.

Now let me go to finite temperature, or more precisely, finite N_t , the number of sites in the temporal direction. Along the β axis, representing the pure glue theory, a deconfinement transition is expected [6]. For an $SU(3)$ gauge group, this transition is expected to be first order. Turning on the fermion hopping, this transition should begin

to move in β , the first effect being an effective renormalization of β down toward stronger couplings. In the process, the transition may soften, and perhaps eventually turn into a rapid crossover rather than a true singularity. In any case, the numerical evidence is for a single transition where both the Polyakov line and the chiral symmetry order parameter undergo a rapid change. The transition region should continue into the (β, K) plane to eventually meet the bulk transition line near $K_c(\beta)$ coming in from strong coupling.

On the weak coupling side of the deconfinement transition, physics is dramatically different. Here as the quark mass goes to zero, we expect chiral symmetry restoration in the thermal æther. In terms of the effective potential, we expect only a single simple minimum. Most importantly, we do not expect any singularity around zero quark mass, with physics depending smoothly around the $(\beta, K) = (\infty, \frac{1}{8})$ point. In other words, we expect the chiral transition at small quark masses to be absorbed into the finite temperature transition. As the hopping continues to increase, the $m \leftrightarrow -m$ symmetry of the continuum theory will play a role, bouncing the deconfinement transition back towards larger β after K passes K_c .

What is less clear is what happens to the finite temperature line as we continue further toward the chiral transitions of the doublers. Here I conjecture that another transition line enters the picture. For small N_t the theory is effectively a three dimensional one, which should have its own chiral transition, possibly somewhere between $K = \frac{1}{8}$ and $K = \frac{1}{4}$. Speculating that the deconfinement transition bounces as well off of this line, but on the opposite side, I arrive at the qualitative finite temperature phase diagram sketched in Fig. (4).

To summarize the picture, at small β and small K we have the usual low temperature confined phase. Increasing K , we enter the Aoki phase with spontaneous breaking of flavor and parity. As β increases, the Aoki phase pinches down into either a narrow point or a single first order line, leading towards the free fermion point at $(\beta, K) = (\infty, \frac{1}{8})$. Before reaching that point, this line collides with and is absorbed in the deconfinement transition line. The latter then bounces back towards larger β . Above the chiral line is a phase nearly equivalent physically with the usual confined phase, just differing in the sign of the light quark masses. Indeed, the only physical difference is via the lattice artifacts of

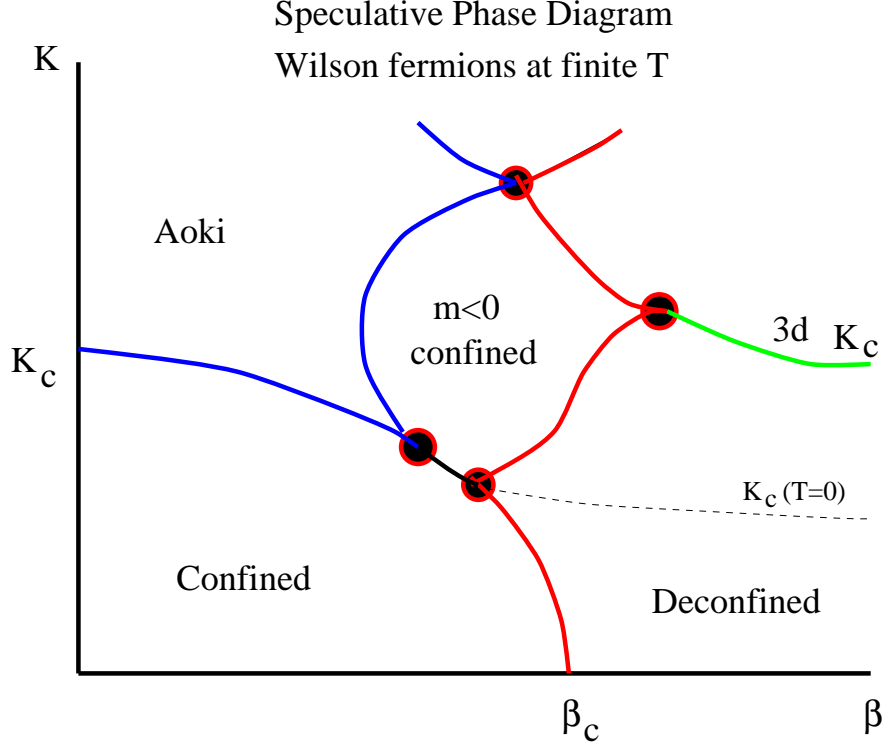


Fig. (4) The conjectured (β, K) phase diagram for finite N_t .

the doublers. Finally, and most speculatively, there may be a three dimensional chiral line coming in from large β which reflects the deconfinement transition back to meet the doubler chiral line heading towards $(\beta, K) = (\infty, \frac{1}{4})$.

This diagram is wonderfully complex, probably incomplete, and may take some time to map out. Given the results presented by Ukawa at this meeting [7], it appears that we may as yet be at too small a value of N_t for the negative mass confined phase to have appeared. As a final reminder, this entire discussion is of lattice artifacts, and other lattice actions, perhaps including various “improvements,” will look dramatically different.

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