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Lattice formulation of the standard model ★

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Abstract

Combining the Kaplan surface mode approach for chiral fermions with added terms motivated by Eichten and Preskill suggests the possibility for a lattice regularization of the standard model which is finite, exactly gauge invariant, and only has physically desired states in its low energy spectrum. The conjectured scheme manifestly requires anomaly cancelation and explicitly contains baryon and lepton number violating terms. Published by Elsevier Science B.V.

From the beginnings of lattice gauge theory, chiral symmetries have been perplexing. The issues revolve around anomalies and fermion doubling. For vector-like theories, such as the strong interactions via gluon exchange, the problems are largely resolved. The standard Wilson [1] approach adds a symmetry breaking term to give all doublers a mass which becomes infinite with the cutoff scale. The approach breaks chiral symmetry rather severely, with the usual current algebra predictions only expected in the continuum limit. While somewhat inelegant, the procedure is well defined and widely adopted.

The situation is more clouded for the full standard model. Here chiral symmetry plays a fundamental role, with neutrinos maximally violating parity. To couple a gauge field, such as the W, to the requisite chiral currents is considerably less straightforward. Among the interesting requirements is the baryon violating process discussed by 't Hooft [2] in the context of topologically non-trivial gauge configurations. As emphasized by Eichten and Preskill [3] and further discussed by Banks [4], a valid lattice formulation must allow for such processes and incorporate terms which violate all anomalous symmetries. Early attempts to include such in the context of a generalized Wilson action met with difficulties [5].

A particularly beautiful feature of the original Wilson lattice theory [6] is its exact local gauge invariance. While one possible approach to the standard model is to break chiral symmetry explicitly, as with the Wilson fermion approach, for the weak interactions this will also break the gauge symmetry, requiring a plethora of counter terms [7]. Our goal is a lattice formulation that keeps all gauged symmetries exact.

A few years ago Kaplan [8] suggested a lattice generalization of an analysis by Callan and Harvey [9] as the basis for a theory of chiral fermions. The approach uses topological defects to bind fermionic

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zero modes [10]. A "domain wall" in five dimensions can naturally bind chiral states. In band theory these modes are known as Shockley [11] states, and arise when the particle states and the Dirac sea are strongly coupled [12]. This approach, and an elegant variation by Narayanan and Neuberger [13–15], have rekindled interest in chiral theories on the lattice. The extension to an extra dimension is also quite reminiscent of anomaly effects in chiral Lagrangian theory [16]. Nevertheless, subtle confusion revolves around making the extra dimension infinite [17,18]. Here we strive to control this limit, providing further support for the approach of Refs. [13–15].

When the extra dimension is finite, the topological defects are naturally paired. For every domain wall there is a mirror defect carrying additional modes. This naturally gives rise to a doubling of species; indeed, this pairing is the minimal amount required by basic theorems [19]. This scheme does provide a promising approach to the chiral symmetries of the strong interactions [20], but for the electro-weak theory gives unwanted "mirror" particles. Here we argue that one can deal directly with Kaplan fermions on a finite lattice, using a variation on the Eichten-Preskill idea to give the mirrors masses of the order of the cutoff.

We start with the standard five-dimensional Wilson fermion theory with hopping parameter sufficiently large that surface modes appear [12]. Our boundary condition is open in the fifth dimension, implementing Shamir's [21] variation on the Kaplan approach (this detail is not essential). We take ordinary space-time dimensions as periodic. We add enough fermionic fields to establish on one four-dimensional face of this system all the desired fermionic states of the standard model, i.e. a strong triplet of weak doublets of quarks and a lepton doublet for each generation. We make no attempt to explain why the real world seems to have three generations, and thus just repeat this structure three times. Unlike in Ref. [20], we put both the left- and right-handed components of the quarks on the same face. We also include spectator right-handed neutrinos on this wall. While these decouple in the standard model, their mirrors are necessary for the removal of other extraneous states.

At this stage we have the fundamental fermions of the full standard model on one interface. However, on the secondary wall in the fifth dimension an unwanted mirror state exists for each desired mode. As usual with the domain wall approach, we couple the four-dimensional gauge fields equally to each slice, and put no gauge field component in the extra direction. The mirror states then couple to the gauge fields with equal strength but opposite parity as the desired fermions.

We want to give the extra states masses comparable to the cutoff scale. We wish to do this without breaking any of the gauge symmetries. This problem is mathematically equivalent to eliminating an extra generation from the standard model; we just have peculiar parity properties. To remove a family is inherently non-trivial because of the 't Hooft process involving baryon decay. The baryon number change in that process is proportional to the number of generations; thus, to eliminate one requires additional baryon violation.

The presence of the 't Hooft process hints at a way to do exactly what we want. Indeed, 't Hooft described the process in terms of an effective interaction vertex. Considering only a single generation at the hadronic level, a member of the proton-neutron doublet can convert to a member of the positron anti-neutrino doublet. In terms of these physical particles, such a mixing is an off-diagonal mass term. To give the particles additional mass, one can artificially enhance this coupling. Our suggestion is to add such a coupling only on the secondary wall, leaving the primary wall bearing all the low energy fermions of the standard model. In essence, we use the Kaplan approach to separate the desired states from their mirrors, and then apply an Eichten-Preskill interaction to generate a mass gap for the mirrors.

The weak interactions generate the product over generations of such vertices only for left-handed helicities. What we do here differs in two respects. First, rather than the product of such terms, we treat the generations independently and add together terms for each. This simplifies the discussion so we can treat each family separately. Second, we add a vertex of this form for each mirror helicity, both left and right. This will generate a mass gap for all mirrors. We place these terms only on the secondary wall of our five-dimensional formalism.

The above discussion is at the level of the physical particles after confinement is taken into account. At another level, the added vertex is actually a four-fermion coupling, mixing anti-leptons with triplets of quark fields. To write the coupling in a compact form, extend the strong $SU(3)$ index to take on a fourth value representing the leptons. We work with a three indexed fermionic field $\psi_{\alpha,i,s}$ where the first index α represents this four-component combination, the second index represents the two components rotating under the $SU(2)$ of the weak interactions, and the final index represents the spinor components of the fermionic field. For a chiral fermion, the last index can be restricted to only two components. Thus there are a total of 16 independent fermionic variables for each generation. Explicitly in terms of the fields for the u and d quarks and the electron neutrino doublet, these fields are

$$\Psi = \begin{pmatrix} u_1^r & d_1^r & u_2^r & d_2^r \\ u_1^g & d_1^g & u_2^g & d_2^g \\ u_1^b & d_1^b & u_2^b & d_2^b \\ \nu_1 & e_1 & \nu_2 & e_2 \end{pmatrix}. \tag{1}$$

Here the subscripts represent the two components of the chiral field, and the superscripts are the internal symmetry indices of the quark confining dynamics (QCD).

The interaction we are interested in is

$$V = \epsilon_{\alpha_1\alpha_2\alpha_3\alpha_4}\epsilon_{i_1i_2}\epsilon_{i_3i_4}\epsilon_{s_1s_2}\epsilon_{s_3s_4} \times \psi_{\alpha_1 i_1 s_1}\psi_{\alpha_2 i_2 s_2}\psi_{\alpha_3 i_3 s_3}\psi_{\alpha_4 i_4 s_4}. \tag{2}$$

We add to our Hamiltonian or Lagrangian a tunable coupling g times $V + V^\dagger$. Separate vertices are used for left- and right-handed fields, although the weak interactions only generate one of these. In particular, in addition to the usual particles of the standard model we include a light right-handed neutrino on the original wall. As the gauge fields do not interact directly with this particle, it decouples in the continuum limit. On the lattice, however, we use its doubler in the mass generation for the doublers of the right-handed quarks. There may exist some variation of Eq. (2) which avoids this spurious particle; this is a subject for future consideration.

The invariance of the antisymmetric tensors ensures that this interaction respects exactly all the desired symmetries of our system. These include the $SU(3)$ of the strong interactions and the weak $SU(2)$ symmetry. The $U(1)$ invariance follows from the neutrality of the vertex. The coupling is also a Lorentz scalar since chiral fermionic fields transform as a rotation by a complex angle, although as usual this symmetry will be broken by the lattice regularization.

That such a vertex can induce a mass gap follows from a strong coupling (g) expansion about the static limit. In this limit each site decouples, giving

$$\int_f \exp\left(g\sum_n (V(n) + V^\dagger(n))\right) = (Cg^8)^N, \tag{3}$$

where the $\int_f$ is a path-integral over fermionic fields, C is a non-zero constant and N is the number of lattice sites on the second wall. The power of eight on g comes from sixteen fermion factors for each of the two helicities, and the vertex is of fourth order. The kinetic terms for the fermions give a perturbation on this result.

In Hamiltonian language, the basic vertex is a matrix operating on a Hilbert space of 2^{16} basis states. We normalize with the conventional anti-commutation relation

$$[\psi_{\alpha_1 i_1 s_1}, \psi^\dagger_{\alpha_2 i_2 s_2}]_+ = \delta_{\alpha_1\alpha_2}\delta_{i_1i_2}\delta_{s_1s_2}. \tag{4}$$

Regarding the components of ψ as destruction operators and taking $H = V + V^\dagger$, we have a somewhat unusual quantum mechanics problem, where fermion number is only conserved modulo four.

The ground state wave function has fermion number vanishing modulo four. It is most easily expressed by applying $V^\dagger$ to the bare vacuum $|0\rangle$, annihilated by $\psi_{\alpha,i,s}$. Define the normalized state $|n\rangle \propto V^{\dagger n}|0\rangle$. Because at most 16 fermions can be created, this sequence terminates at $n = 4$. The Hamiltonian closes on this set, giving a 5 by 5 matrix problem. The ground state

$$|E_0\rangle = \frac{\sqrt{39}}{26}(|0\rangle + |4\rangle) - \tfrac{1}{2}(|1\rangle + |3\rangle) + \frac{\sqrt{65}}{13}|2\rangle \tag{5}$$

has energy $E_0 = -16\sqrt{78} = -141.308\ldots$ and is nondegenerate. This state is a singlet under both the strong and weak gauge symmetries. As expected, it mixes states of different baryon and lepton number.

Similar manipulations give the first excited state, which turns out to be in the sector mixing states with fermion number 2 mod 4. It has energy $E_1 = -8\sqrt{122 + 10\sqrt{97}} = -118.79\ldots$. This energy represents a multiplet of non-singlet states.

The strong coupling approach starts with each site in the ground state. Treated as a perturbation, the fermion kinetic terms allow hopping between adjacent sites. This will excite the two sites involved, requiring a finite energy. That energy represents a gap in the spectrum, corresponding to the existence of only massive states.

The enhanced vertex should not induce a spontaneous breaking of one of the gauge symmetries in the problem. The unique ground state for the strong coupling expansion shows that this does not happen as long as the four-fermion coupling is sufficiently large compared with the kinetic term.

For our scheme to work, the added coupling must not drastically interfere with the nature of the heavy states in the fifth dimension. Ref. [22] showed such a difficulty with using an infinite Higgs coupling on the secondary wall. If we do take our added coupling to infinity, the last slice in the extra dimension decouples, giving an effective theory with one less slice. This returns us to the starting model with unwanted mirrors. To avoid this we must keep the coupling finite but large enough to apply the above strong coupling analysis on the low energy states. Ref. [22] suggests that a phase with massless mirrors might persist for a finite range of coupling below infinity. If that happens here as well, we must appeal to a hierarchical continuum limit, adjusting the scale of the extra term to be small compared to the scale of the heavy states, but large compared with the weak scale. Here is the weakest point in our argument; a superstrong coupling phase of massless mirrors and a spontaneously broken region at intermediate coupling could possibly squeeze out our desired phase of strongly coupled massive mirrors. Such a situation would cast serious doubts on any construction of chiral gauge theories.

A four-fermion vertex can generally be broken into fermion bilinears interacting with an auxiliary scalar field. Such Yukawa like models have been extensively discussed in the past, particularly in the quest for a chiral fermion theory [23]. These studies show a rather rich phase structure. We want to place the extra wall in a strong coupling phase with a mass gap but not displaying any spontaneous symmetry breaking. Such is sometimes referred to as a paramagnetic strong coupling phase. For our purposes we are not interested in a continuum limit of this Yukawa model; indeed, we want no light particles remaining on the extra wall as the lattice spacing goes to zero. Meanwhile, we always keep the original wall in the weakly coupled phase with light chiral fermion states. If the arguments of Ref. [22] for a massless phase at ultra-strong coupling hold for our model as well, then there is a second paramagnetic strong coupling phase which we must avoid.

Our scheme gives an intuitive description of anomalous currents, generalizing the discussion in Ref. [12]. When a topologically non-trivial gauge configuration induces baryon flow out of the primary wall of our five-dimensional system, this current continues to the secondary wall where baryon number is strongly violated. The latter wall acts as an unusual mirror, reflecting the baryons back as leptons. The lepton flow returns to the primary wall and cancels the lepton number also coming from the topological transition. Because there is a mass gap on the secondary wall, it acts as a perfect mirror, giving no additional factors to the usual tunneling expression. In this process the difference of baryon and lepton number is exactly conserved, just as in the usual continuum standard model.

Anomaly cancelation is essential to our picture. In the standard model both the quarks and the leptons must be present. Otherwise this gauge invariant vertex does not exist. Even though we have strong baryon violation on one wall, this need not induce unacceptably large baryon decay for the physical particles. Communication between the two domain walls is exponentially suppressed for all but anomalous currents. The same small factors as in the usual continuum treatment [2] suppress the latter.

In some sense the theory still has doublers, but we use them. The extra term converts the lepton mirrors to composite three quark states, and the quark doublers to lepton diquark combinations. Also note that the chiral partner of a given state is a convention related to the particular gauge field being considered. From the standpoint of the strong interactions, the left- and right-handed quarks are partners of each other. For the weak interactions the matching of particles with doublers is most natural in the above twisted manner. At the level of physical particles, the interpretation is simpler: the doubler of the left-handed electron is the right-handed anti-proton. Both have the same charge, are singlets under strong $SU(3)$ and are members of weak doublets.

Our proposal makes no use of the Higgs mechanism usually used to generate particle masses. Indeed, since our starting point is exactly gauge invariant, the Higgs generation of physical fermion and weak boson masses need only be applied in the standard manner at the last stage. This might raise new non-perturbative issues, but goes beyond the subject of this paper. Since the mirror world has all anomalies properly canceled, it should decouple from the low energy physics of the normal standard model in the continuum limit.

Two-dimensional models are often suggested as a testing ground for chiral theories. For example, Ref. [27] studies the electrodynamics of a right moving charge two fermion canceling anomalies with four left movers of unit charge. This model introduces a tricky twist into the scheme we propose. In particular, the analog of the 't Hooft vertex contains a derivative. This requires the mixing of neighboring sites, and appears to complicate the generation of the desired mass gap on the secondary wall. For this case we are unsure whether a deficiency might lie in our approach. Indeed, the extent to which special features of the standard model are crucial to our approach remains an open question.

While we have used the Kaplan approach to set up the initial model, presumably one could also work directly with simpler mirror fermion models [28]. Another direction might be to combine these ideas with the use of a shift symmetry to separate the doublers as in [29]. Nevertheless, the use of an extra dimension to separate the problem seems conceptually useful.

If successful, this approach would further justify several alternative chiral fermion techniques. For example, the overlap formalism [13] effectively represents an infinite extra dimension and ignores the secondary wall. Eliminating of that wall in an exactly gauge invariant manner would support their conclusions as well as several other schemes that involve an additional infinity [24–26]. Our approach is perhaps cleaner in that gauge invariance is exact, all infinities are eliminated, and the requirement of anomaly cancelation is manifest.

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