Are magnetic monopoles hadrons?

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The charges of magnetic monopoles are constrained to a multiple of 2π times the inverse of the elementary unit electric charge. In the standard model, quarks have fractional charge, raising the question of whether the basic magnetic monopole unit is a multiple of $2\pi/e$ or three times that. A simple lattice construction shows how a magnetic monopole of the lower strength is possible if it interacts with gluonic fields as well. Such a monopole is thus a hadron. This is consistent with the construction of magnetic monopoles in grand unified theories.

Some time ago G. 't Hooft [1] discussed an amusing puzzle involving the quantization of magnetic monopole charges. As is well known from Dirac [2], quantum mechanical consistency for a hypothetical monopole requires its charge to be quantized in units of $\frac{2\pi}{e}$, where e is the smallest non-zero charge of any existing particle. The puzzle arises on considering quarks, which have charges that are third integer fractions of the charges on free particles. Is the quantization unit $\frac{2\pi}{e}$ or thrice that?

The resolution [1] is entwined with the phenomenon of confinement and the strong force. Indeed, the minimum charge is $\frac{2\pi}{e}$, but to have this value, a magnetic monopole must also interact with gluon fields. Here I return to this old result and discuss it in simple lattice gauge language.

I begin the discussion with a description of how a static magnetic monopole is formulated in pure U(1) lattice gauge theory. Since it is static, I ignore the time direction, along which all links and plaquettes are unmodified. The following structure is repeated on each time slice.

To start, use a "Dirac string" to bring in from infinity the net flux emerging from the monopole. For the gauge fields, there is a phase U_l associated with each link on our lattice. The gauge action is constructed by multiplying links around plaquettes $U_p = \prod_{l \in p} U_l$.

Ordinarily the action just adds the real parts of these plaquette variables together. But with the Dirac string present, we insert an additional phase on plaquettes pierced by the string and take

$$S = \sum_p \mathrm{Re}(U_p e^{i\phi_p}). \qquad (1)$$

Here ϕ_p is the external flux brought through the plaquette and vanishes except along the string ending at the monopole location.

So, to have a monopole centered in a cube at the origin, one can run the string down the negative z axis. Then $\phi_p = 0$ except on xy plaquettes at $x = y = 0$, $z \leq 0$. For those plaquettes, ϕ_p is a constant and represents the monopole strength. This is illustrated in Fig. (1).

The route taken by the Dirac string can be changed by simple changes of variables. For example, if we absorb the factor of $e^{i\phi_p}$ on some plaquette into the measure for one of the links on its side, we reroute the Dirac string through two neighboring cubes, as illustrated in Fig. (2).

Note that if the monopole has strength $\phi = 2\pi$, the action is equivalent to the pure gauge theory without the monopole present. As interpreted by Degrand and Toussaint [3], the compact theory has intrinsic monopoles of strength 2π which completely screen the external monopole.

So far I have been discussing the theory without dynamical charges. Now include an electrically charged fermion of intrinsic charge e. When this fermion hops between neightbors along link l, its wave function
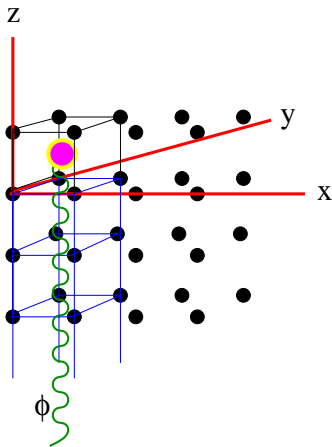
Figure 1. The construction of a static monopole at the origin. The monopole flux is brought in along a Dirac string parallel to the negative z axis.

Figure 2. The path taken by a Dirac string is unphysical since it can be rerouted by simple changes of variables on the lattice links.

picks up a phase $(U_l)^e$. Unlike the pure gauge situation, the Dirac string with arbitrary strength becomes observable. When the fermion hops around it, it picks up an extra phase $e^{ie\phi}$. Only if $e\phi = 2\pi n$ is the string unobservable. This is the Dirac quantization condition.

Now I turn to discuss another type of string, a Z_3 string in the quark confining dynamics of the strong interactions. The complex phase $e^{2\pi i/3}$ is an element of $SU(3)$. It commutes with all $SU(3)$ elements, and generates the center of the group. We can use this element to generate strong "monopoles" in the pure $SU(3)$ gauge theory. This is done in direct analogy with the above construction of $U(1)$ monopoles; a flux of strength $e^{2\pi i/3}$ is brought in along a new "Dirac string" by inserting this Z_3 factor into plaquettes pierced by the string. This is illustrated in Fig.(3).

In the pure glue theory, the path of this string can also be changed by simple changes of variables. Thus the path is, as in the $U(1)$ case, unphysical. However, when quarks are introduced into the theory, the Dirac string becomes observable due to the extra Z_3 factor they encounter on circumnavigating the string.

Next consider the combined $U(1) \times SU(3)$ model. We have on each link l both a phase factor $U_l \in U(1)$ and a a strong fac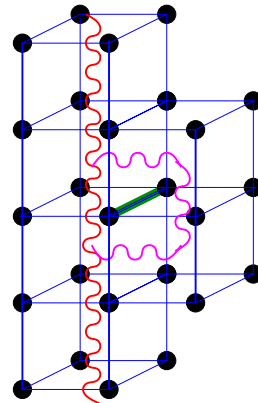tor $V_l \in SU(3)$. The $U(1)$ and the $SU(3)$ fields do not directly interact with each other, but they are coupled via the quarks, which interact with both.

In the coupled theory, consider superposing the $U(1)$ and the $SU(3)$ strings, as sketched in Fig. (4). Suppose the $U(1)$ strength is $\phi = 2\pi/e$ so as to make the string invisible to electrons. Of course, the electrons are neutral with respect to gluons and do not interact with the $SU(3)$ string.

Quarks, interacting both with gluons and photons, do interact with both strings. For example, the down quark gets factor of $e^{-2\pi i/3}$ from the $U(1)$ string. But it also gets a factor of $e^{2\pi i/3}$ from the Z_3 string. We have a remarkable phase cancellation between these factors, and the combined string is not observable. Note that this cancellation locks in the quark charges, *i.e.* it only works if $e_d = -e/3$ modulo full units of electric charge. This connection with the quark electromagnetic charge is automatic in grand unified theories [4,5].

Since this minimally charged monopole involves the $SU(3)$ fields, it is strongly interacting. In particular, it will disturb the surrounding gluonic fields. For example, the plaquette expectation value will be modified in the vicinity of the monopole. This can be easily seen in the strong coupling expansion, where this correlation appears in a diagram involving tiling a
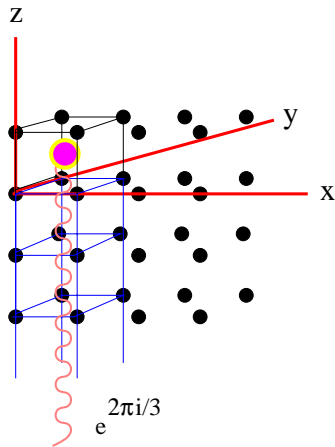
Figure 3. In analogy with the $U(1)$ monopole, a Z_3 monopole can be constructed for the $SU(3)$ gauge theory of the strong interactions.
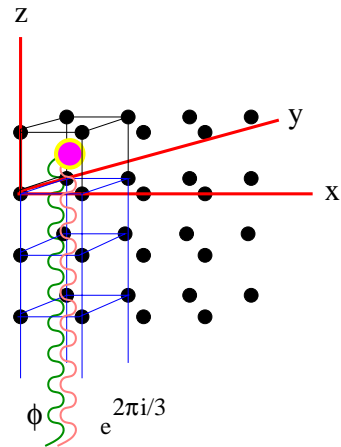

Figure 4. Superposing $U(1)$ and $SU(3)$ strings allows a construction invisible to both electrons and quarks.
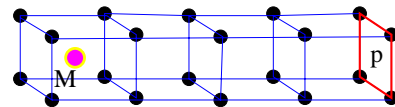

Figure 5. A plaquette near a monopole has its expectation value modified. In the strong coupling limit, this modification appears in a diagram similar to that used to calculate the glueball mass.

tube contaning the given plaquette and also surrounding the monopole. This diagram is shown in Fig.(5), and is quite similar to the strong coupling diagram for the glueball mass M_g. The leading behavior takes the form

$$\langle U_p \rangle_M - \langle U_p \rangle_0 \sim \beta^{4L}(e^{2\pi i/3} - 1) \sim e^{-M_g L}. \quad (2)$$

To this order the screening length for the hadronic fields about a monopole is given by the glueball mass. At higher orders diagrams involving pions will give the longest range behavior.

To summarize, magnetic monopole charges are quantized in units of $2\pi/e$, not $3 \times 2\pi/e$. But to have this value, a minimally charged monopole must interact strongly. To leading order, the magnetic glue screening length is controlled by glueball mass. Finally, this scheme works only if quark charges are fixed at their usual values, as in grand unified theories. While this hints at unification, it does not require such.

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