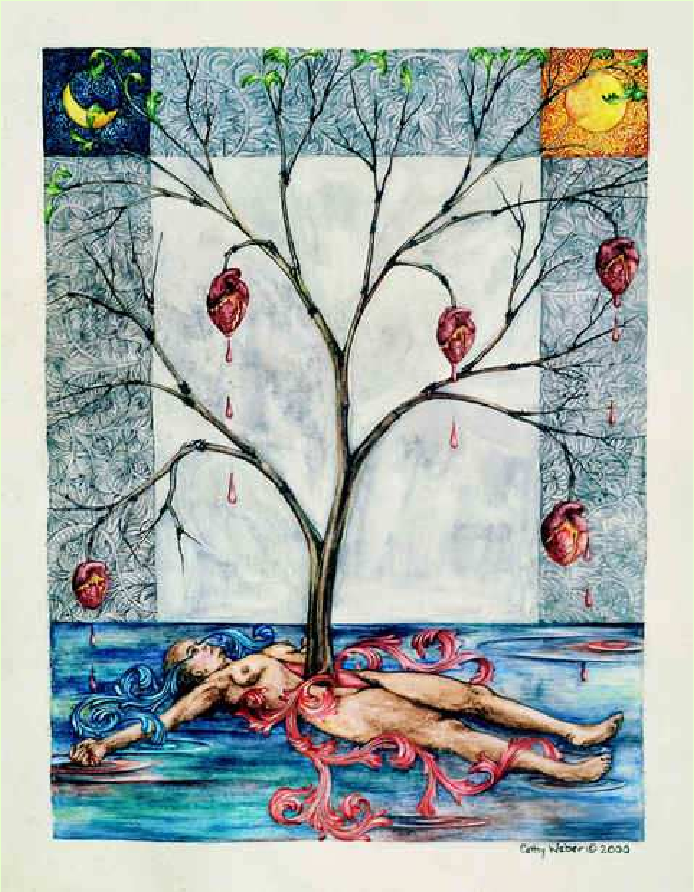

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Why rooting fails

Michael Creutz

Brookhaven National Laboratory

MC, lattice '06: what staggered rooting gets wrong

- excess symmetry gives an incorrect mass dependence

This presentation: why

- strong chirality averaging gives incorrect 't Hooft vertex

Outline

Why rooting naively looks sensible

- determinant as a sum over loops
- Dirac eigenvalues

Staggered review

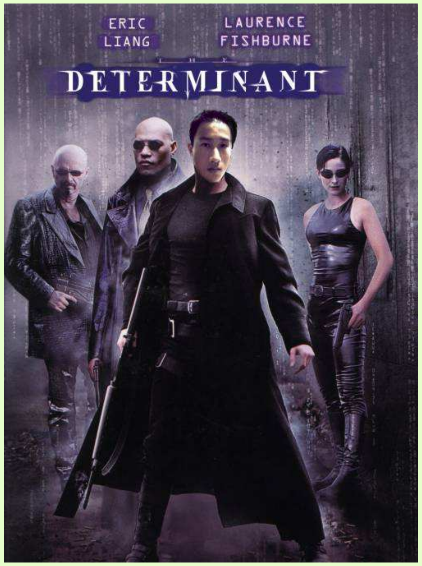
Where things go awry

- taste chirality mixing
- eigenvalue flow must break taste symmetry

The ’t Hooft vertex as the essence of the problem

- strongly couples all tastes
- excess symmetry prevents proper rooting

The fermion determinant



Determinant sums over permutations of matrix rows

- each permutation factors into cycles
- each cycle represents a fermion loop
- doubling by factor of 4 in staggered fermions
- each loop counted 4 times too much

$|D| \rightarrow |D|^{1/4}$ multiplies each loop by $1/4$

Rooting reproduces correct perturbative expansion

Eigenvalues

Diagonalize $|D|$

- $|D| = \prod_i \lambda_i$
- λ_i appear to arrange into taste quartets

Rooting should select one eigenvalue from each quartet

- reasonable as long as quartets cleanly separated
- observed to improve as lattice spacing decreases

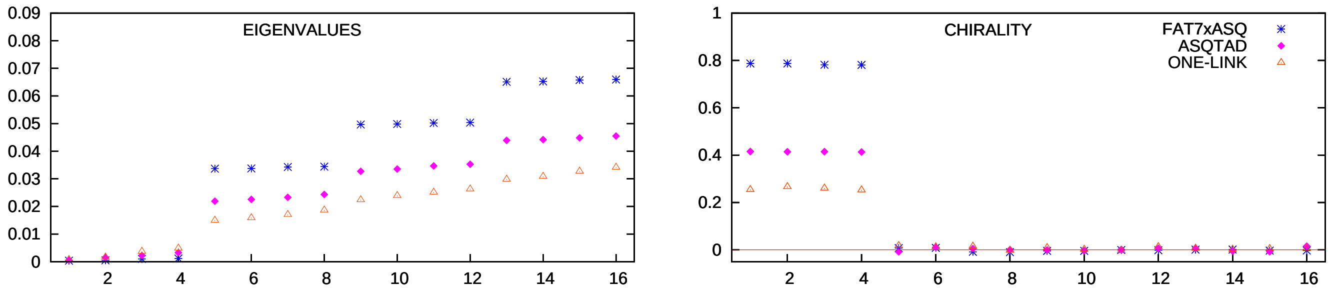
Taste symmetry essential

- Bernard, Golterman, Sharpe, Shamir

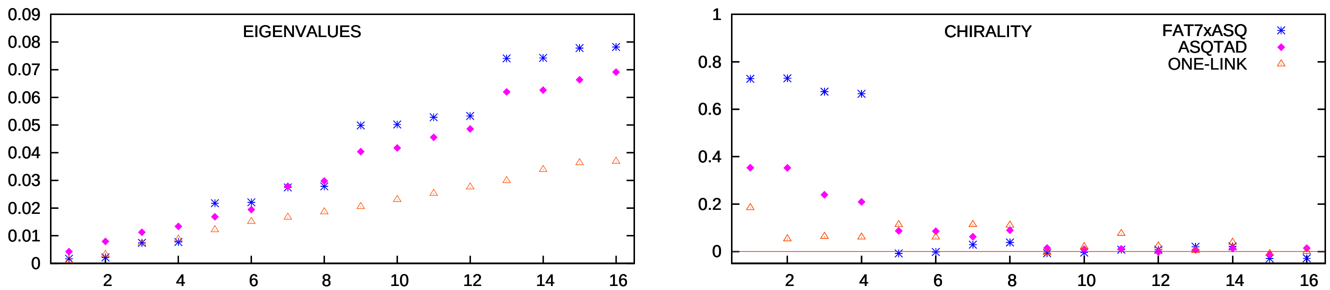
Beware of hidden tastes

Follana, Hart, Davies, Mason: hep-lat/0507011

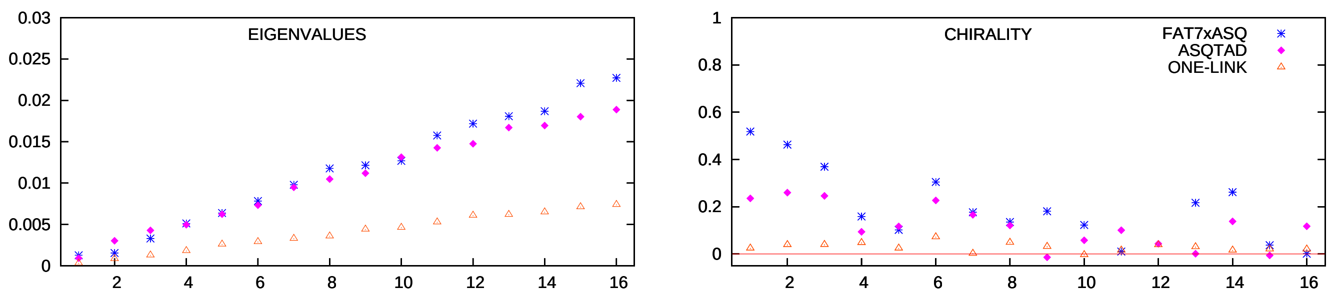
Improved Glue (a ≈ 0.077 fm, V = 20^4)



Improved Glue (a ≈ 0.093 fm, V = 16^4)

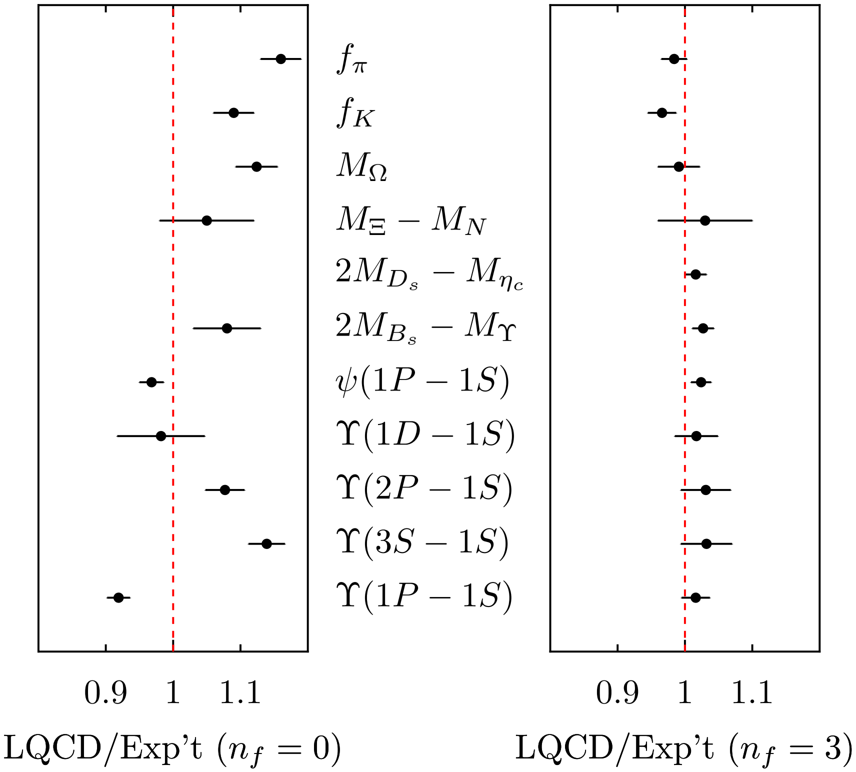


Wilson Glue (a ≈ 0.1 fm, V = 16^3 x 32)



Staggered fermions have had astounding numerical success

- proven to be a good approximation

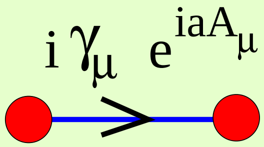


Phys.Rev.Lett.92:022001,2004

Review of staggered fermions

Start with naive fermions, $i\gamma_\mu$ for each hop in direction μ

- $\gamma_\mu p_\mu \to \gamma_\mu \frac{\sin(ap_\mu)}{a}$

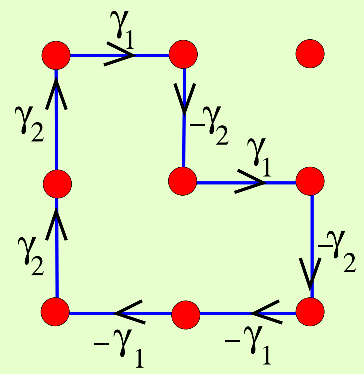


Propagator has poles whenever components of momentum are 0 or π/a

- 16 "doublers"
- different chiralities since $\frac{d}{dp}\sin(p)|_{p=\pi} = -1$
  - helicity projectors $(1 \pm \gamma_5)/2$ depend on doubler

Exact naive chiral symmetry maintained

- actually a flavored symmetry of the doublers



In a closed fermion loop

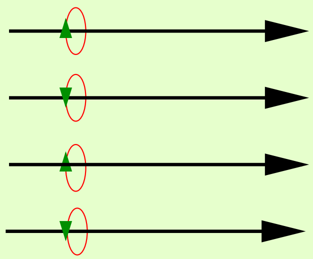
- each factor of γ_μ appears an even number of times
  - product proportional to the identity
  - four spinor components of ψ are independent
- exact $U(4) \otimes U(4)$ chiral symmetry   Karsten and Smit (1981)
  - anomaly OK: symmetry not $U(16) \otimes U(16)$

Staggered fermions divide out this $U(4)$ symmetry

Project out one component per site $\psi \to P\psi$

$$P = \frac{1}{4}\left(1 + i\gamma_1\gamma_2(-1)^{x_1+x_2} + i\gamma_3\gamma_4(-1)^{x_3+x_4} + \gamma_5(-1)^{x_1+x_2+x_3+x_4}\right)$$

- reduces 16 doublers to 4
- exact chiral symmetry remains
- OK: still a flavored symmetry among the doublers
  - tastes not equivalent
  - two of each chirality



Rooting

- replace fermion determinant $|D|$ with $|D|^{1/4}$
- hope to reduce effect of four doublers to one
- BUT: $U(1)$ chiral symmetry remains
  - flavored symmetry without flavors?

Alberta resides under a boutique in the Yucatan

Comment on chiral symmetry

An $SU(N_f) \otimes SU(N_f)$ symmetry of the massless theory

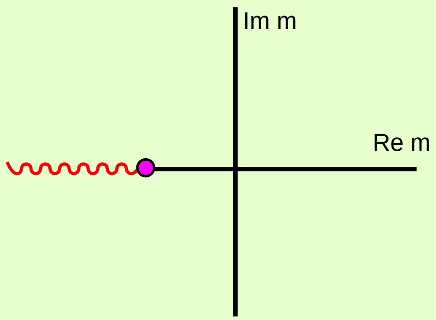
- spontaneously broken, explains lightness of pions

Also a symmetry of the massive theory in parameter space

- mass term $\frac{1}{2}(\overline{\psi}_L M \psi_R + \overline{\psi}_R M^\dagger \psi_L)$
- physics invariant under $M \to g_L^\dagger M g_R$ where $g_L,\ g_R\ \in\ SU(N_f)$

$M \to e^{i\theta} M$ not a symmetry

- changes the strong CP violating angle
- $N_f^2 - 1$ Goldstone bosons



One flavor QCD should have no chiral symmetry

- analytic in m at $m = 0$

MC, lat06

The problem

Before rooting we have one exact chiral symmetry

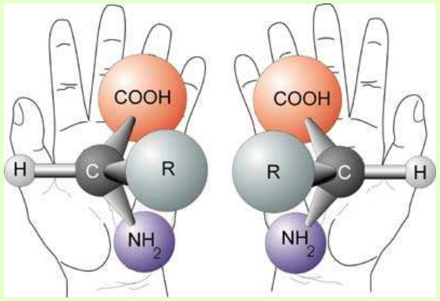
- actually a non-singlet symmetry
  - $\gamma_\mu \frac{d}{dp} \sin(p)|_{p=\pi} = -\gamma_\mu$
  - different tastes use different gamma matrices
- two “tastes” of each chirality

Index theorem for unit gauge field winding

- one approximate zero mode for each “taste”
- two left handed
- two right handed

Rooting averages over these

- not the single chirality of the target theory

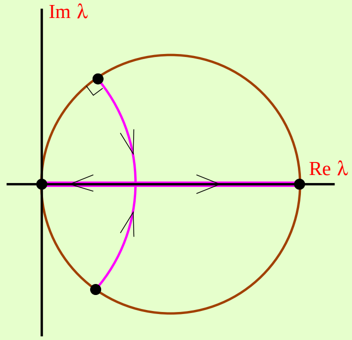


The staggered projection

- $P\gamma_5 = \gamma_5 P = (-1)^{x_1+x_2+x_3+x_4} P$
- $\gamma_5 \to (-1)^{x_1+x_2+x_3+x_4}$ independent of gauge field
- remains traceless

Approximate zero modes must come in opposite chirality pairs

- unlike "continuum"
  - modes can be lost or appear at infinity
- unlike Wilson
  - modes paired with heavy doublers
- unlike overlap
  - opposite chirality modes at $\lambda = 2$
  - $\mathrm{Tr}\hat{\gamma}_5 = 2\nu$



MC, lat02

Eigenvalue flow

Smooth gauge field with zero winding number

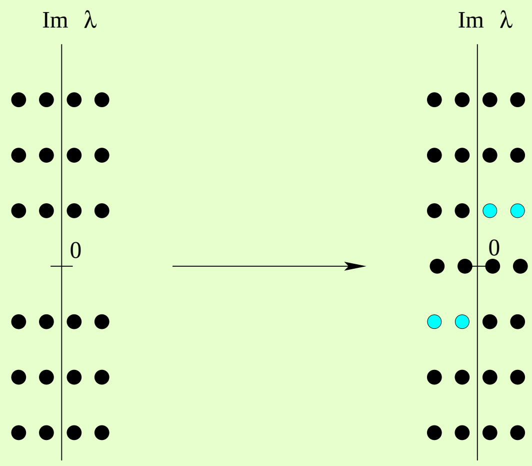
- unrooted eigenvalues in quartets

Smooth gauge field with unit winding number

- one quartet near zero

Transiting topology forces breakup of quartets

- two eigenvalues come from above, two from below
- forces a mismatch distributed among non-zero eigenvalues



Zero modes and the 't Hooft vertex

Insert a small mass into the path integral

$$Z = \int dA \, e^{-S_g} \prod (\lambda_i + m)$$

As m goes to zero any configurations involving a $\lambda = 0$ drop out

- are "instantons" irrelevant in the chiral limit?

No: add sources $\eta, \overline{\eta}$

$$Z(\eta, \overline{\eta}) = \int dA\, d\psi\, d\overline{\psi}\, e^{-S_g + \overline{\psi}(D+m)\psi + \overline{\psi}\eta + \overline{\eta}\psi}$$

- integrate out fermions

$$Z = \int dA\, e^{-S_g + \overline{\eta}(D+m)^{-1}\eta} \prod (\lambda_i + m)$$

If source overlaps with a zero mode eigenvector $(\psi_0, \eta) \neq 0$

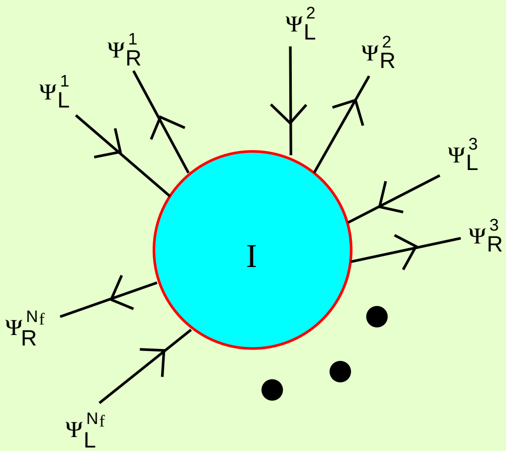
- $1/m$ in source term cancels m from determinant

Instantons do drop out of Z

- but survive in correlation functions

With multiple flavors

- need source factor from each flavor: “ ’t Hooft vertex”



N_f flavors give $2N_f$-fermion effective interaction

- non-perturbative
- represents the anomaly
- high dimensions compensated by Λ_{qcd}

Four staggered tastes: an octilinear interaction $\sim (\overline{\psi}\psi)^4$

- strongly couples all tastes
  - even in the continuum limit
- chiral symmetry OK since two tastes of each chirality

One flavor: need a bi-linear interaction $\sim \overline{\psi}\psi$

- inconsistent with the exact chiral symmetry
- forbidden in the rooted theory
  - wrong RG mass flow

But if $|D| \sim m^4$ then $|D|^{1/4} \sim m$

- each taste gives the correct vertex
- measure it for one taste
  - ignore the others

No: the vertex strongly couples all tastes

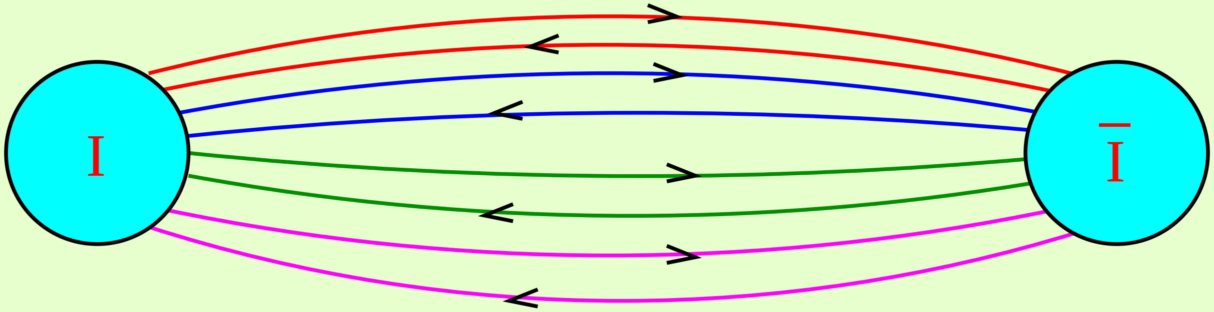
- strength $\sim \frac{1}{m^4} * (m^4)^{1/4} \sim \frac{1}{m^3}$
- unphysical singularity at $m = 0$
  - scale set by Λ_{qcd}
  - no $O(a)$ suppression
  - high gluon momenta not involved

All tastes contribute in intermediate states

- incorrect multi-instanton interactions

One contribution to instanton/anti-instanton interaction:

- affects $\langle F\tilde{F}(y) \qquad F\tilde{F}(x)\rangle$
- exchange all four tastes



Unrooted: $m^4 \quad * \quad m^{-8} \quad * \quad m^4 \quad \sim$ const

- x dependence from "zero mode" overlap

Rooted: $m \quad * \quad m^{-8} \quad * \quad m \quad \sim m^{-6}$

Target: $m \quad * \quad m^{-2} \quad * \quad m \quad \sim$ const

- Pauli principle: only one exchange possible

Questions

Is a square root better?

- doublers do occur in equivalent pairs
- residual chiral symmetry allowed

Are massive quarks better behaved?

- wrong chiral behavior unimportant?

} Accurate 2+1 theory?

Can counterterms fix things?

- replace unphysical singularity with the correct vertex
- requires tuning
- non-local?

Cancel extra tastes with bosonic “ghosts”?

- need a chiral formulation

Conclusion

Rooting is justified perturbatively

- accurate for many physical quantities

Rooting does not generate the correct ’t Hooft vertex

- dangerous for non-perturbative physics in singlet channels